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Principles of Distributed Computing
Exercise 6: Sample Solution

1 Shared Sum

In the following, let X (initialized to 0) always denote the shared register used to hold the sum $x = \sum_{i=1}^{n} x_i$, and assume that all x_i (and thus also x) are initially 0. Denote by Δx_i the amount by which x_i is changed by process p_i at some time, i.e., if $x_i := x'_i$ is assigned by p_i, then $\Delta x_i = x'_i - x_i$.

a) To update x, p_i calls fetch-and-add$(X, \Delta x_i)$. Therefore, X changes exactly the same as x_i and holds the correct value. Since no process has to wait or retry, we have neither lockouts nor deadlocks. A simple read on X (or fetch-and-add$(X, 0)$) gets the current value of x.

b) An update is done by the following code:

$x := X$
while not compare-and-swap$(X, x, x + \Delta x_i)$ **do**
  $x := X$
end while

The loop is left after X changed by Δx_i exactly once, thus the code is correct. Again, x can be obtained by a simple read. Since the compare-and-swap may only fail if another process p_j changed the value of X between p_i reading it and calling compare-and-swap, there is no deadlock. However, other updates may delay a change by some p_i indefinitely, hence lockouts are possible.

c) A write is implemented by

$x :=$ load-link(X)
while not store-conditional$(X, x + \Delta x_i)$ **do**
  $x :=$ load-link(X)
end while

and is correct for the same reasons as in **b)**. Reads are again simple. Again, deadlocks are impossible since the store-conditional may only fail if something has been written to the register beforehand, but lockouts may occur.

d) It can be done. We use a special encoding on X. Either it stores a regular value and $\perp$ (i.e., $(x, \perp)$) or the value and an additional identifier identifier $id(i)$ of a process p_i. A node will effectively acquire a lock on X by writing its ID to X and only afterwards write its update to X.

When x_i is changed, p_i executes

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while true do
  (x, id) := X // simple read
  (x, id) := compare-and-swap(X, (x, ⊥), (x, id(i))) // try to lock X with own ID
  if id = id(i) then
    X := (x + Δx_i, ⊥) // regular write, but compare-and-swap would also do
    break
  end if
end while
```

Because writing by compare-and-swap works only if the second argument equals the value of the register, once a process "locks" X with its identifier, no other process may do so until the same process performs the write enclosed in the if-condition. Thus, this write happens exactly if the compare-and-swap was successful. The only reason to check the identifier by an if-statement rather than using compare-and-swap again is that we need to ensure that the process leaves the loop after changing X by Δx_i. On the other hand, the while loop can only be left after a succesful write, thus X is updated correctly. Reads are again plain reads.

As before, the solution is free of deadlocks: At least one process can write, because after each write the ID part of X contains $\perp$, i.e., one process will succeed in "locking" X. As in **b)** and **c)**, the solution is prone to lockouts.