Distributed Quantum Computing

Using the most precise description of our world to solve some of the hardest problems

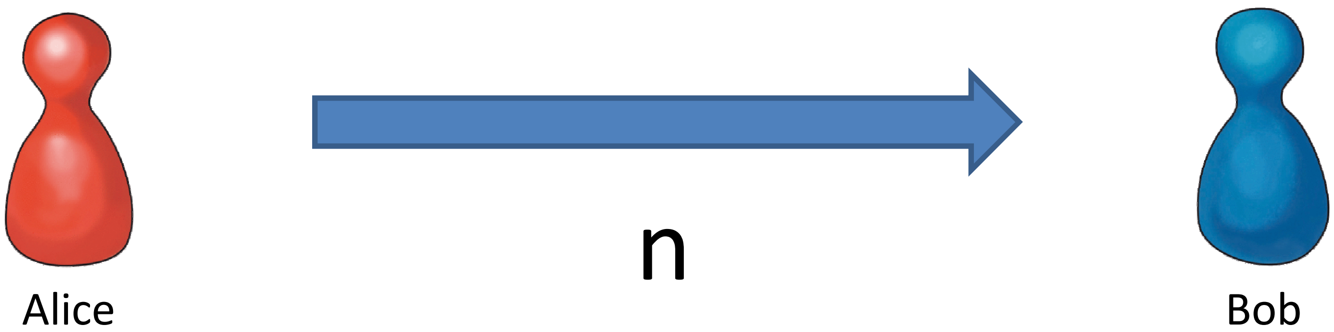
by Matthias Egli

Based on the paper „Distributed Quantum Computing“ by Harry Buhrman and Hein Röhrig

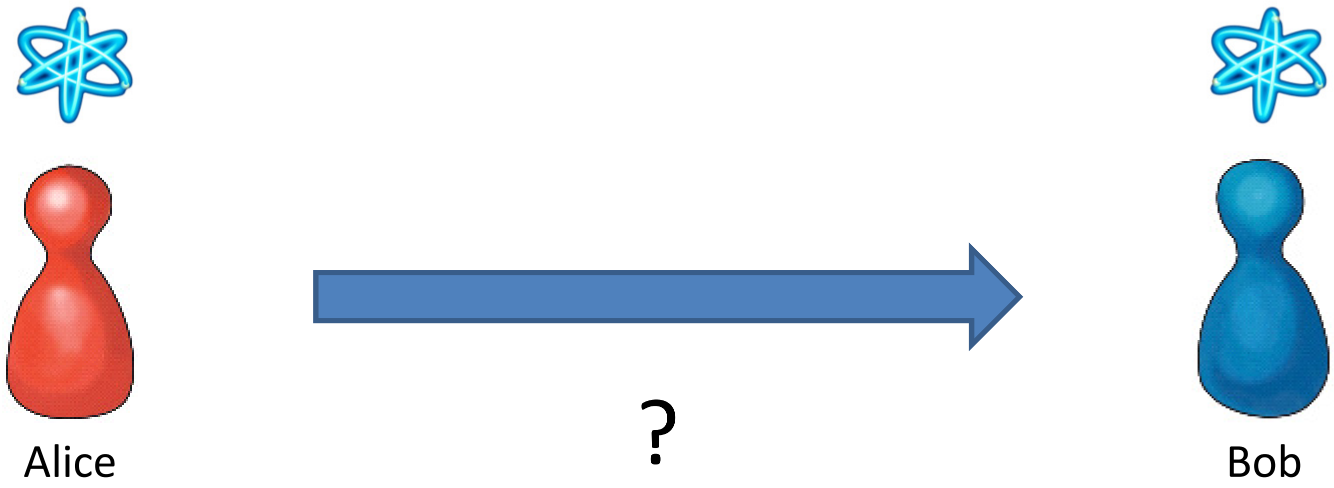
These slides are just an addition to the math shown on the blackboard

Motivation

Communication Complexity



Communication Complexity



Communication Complexity

	Classical Computer	Quantum Computer
Disjointness	n	$\sqrt{n}$
Equality (Promise)	n	log(n)

Speedup with Quantum Computers

	Classical Computer	Quantum Computer
Factorization	a^n	$\log(n)$
Search	n	$\sqrt{n}$
Deutsch-Jozsa	n	1

The qubit $\alpha|0\rangle + \beta|1\rangle = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$

Superposition $\alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle$

Quantum Gate $H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

Entanglement $\alpha|00\rangle + \beta|11\rangle$

Basics

Deutsch-Josza Algorithm

$$f : \{0,1\}^n \rightarrow \{0,1\}$$

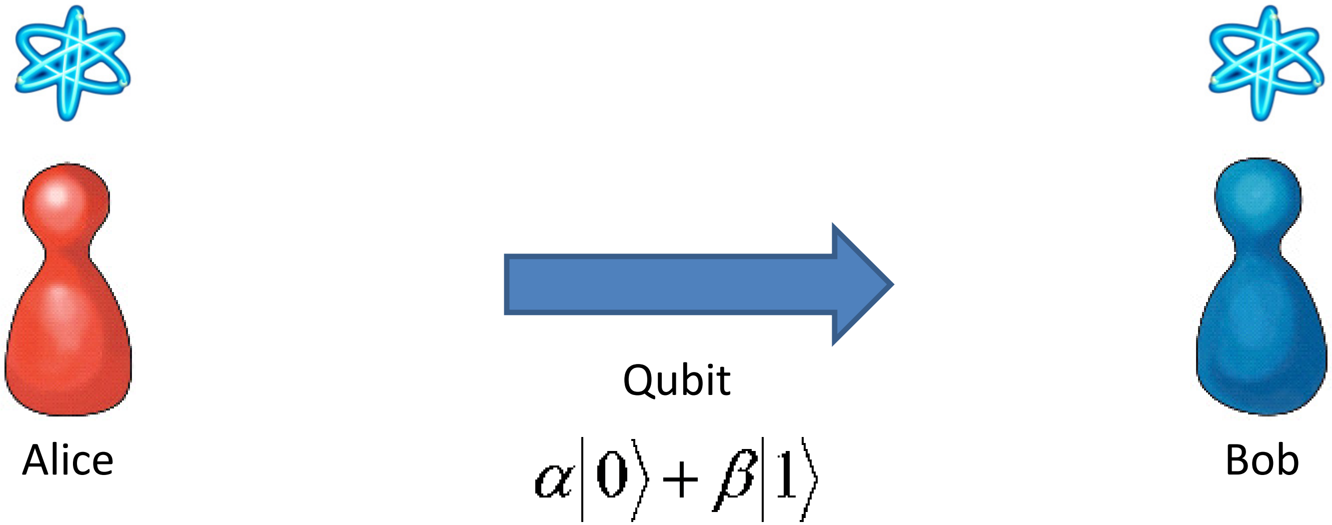
Promise:
Function is constant or balanced

Deutsch-Josza Algorithm

n	Classcial Computer	Classical randomized	Quantum Computer
2	2	1	1
n	n	Log(n)	1

Communication

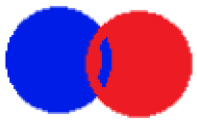
Communication



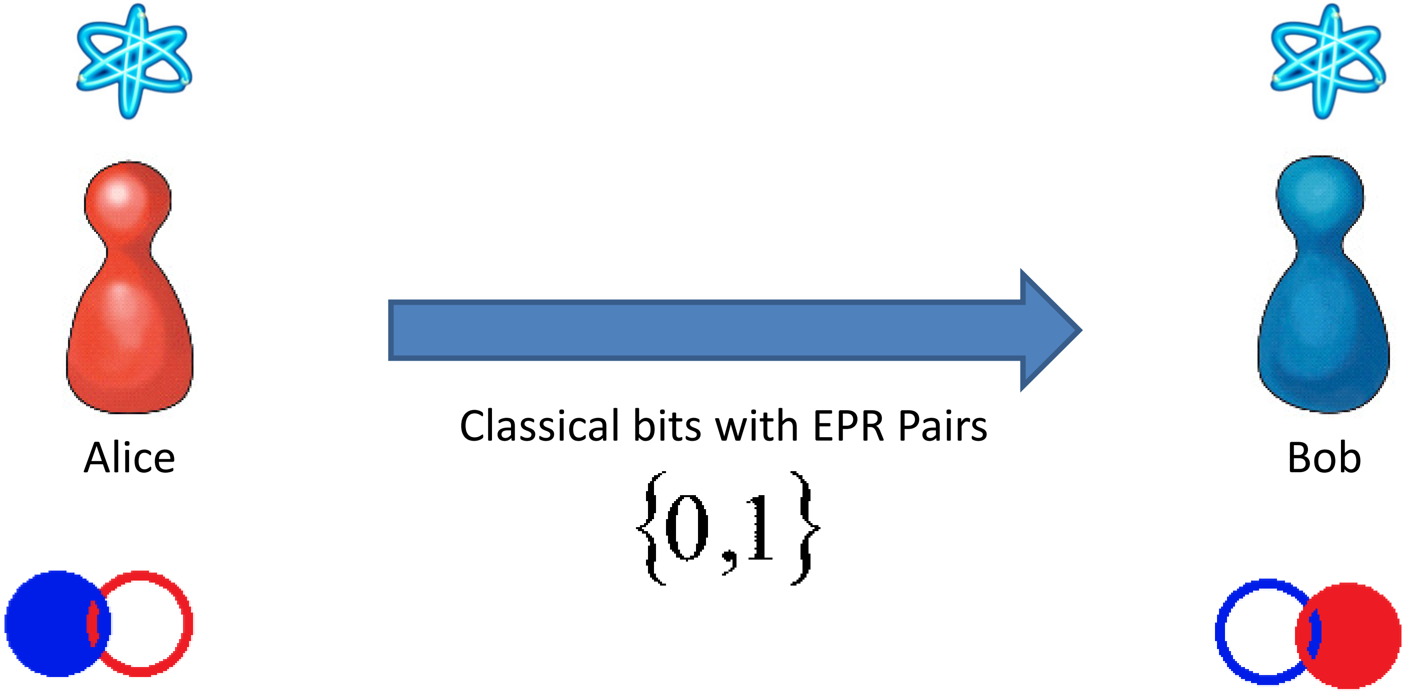
Communication

Entanglement

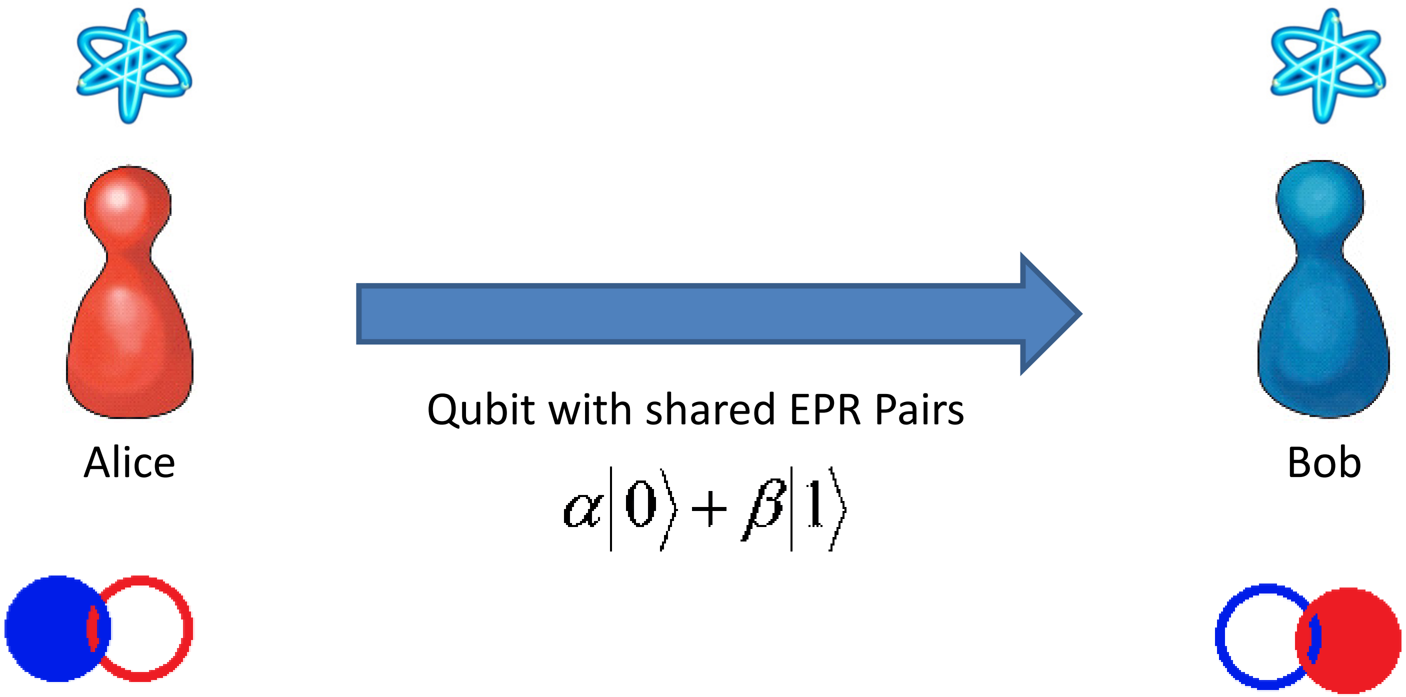
$$\alpha|00\rangle + \beta|11\rangle$$



Communication

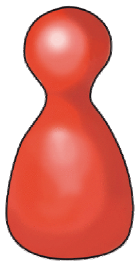


Communication



Communication Types

Deutsch-Jozsa Communication



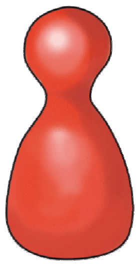
Alice

Bob

X: 00101100

Y: 00101100

Deutsch-Josza Communication



Alice

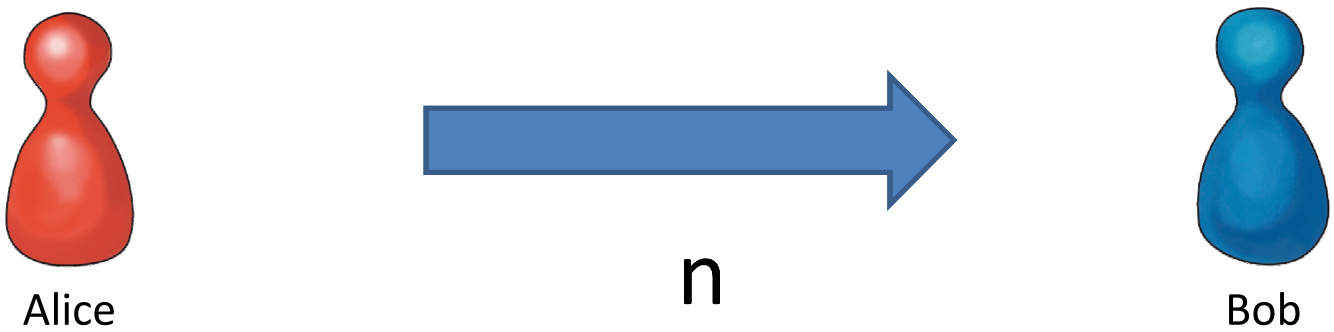
Bob

X: 00101100

$X = Y$?

Y: 00101100

Deutsch-Josza Communication

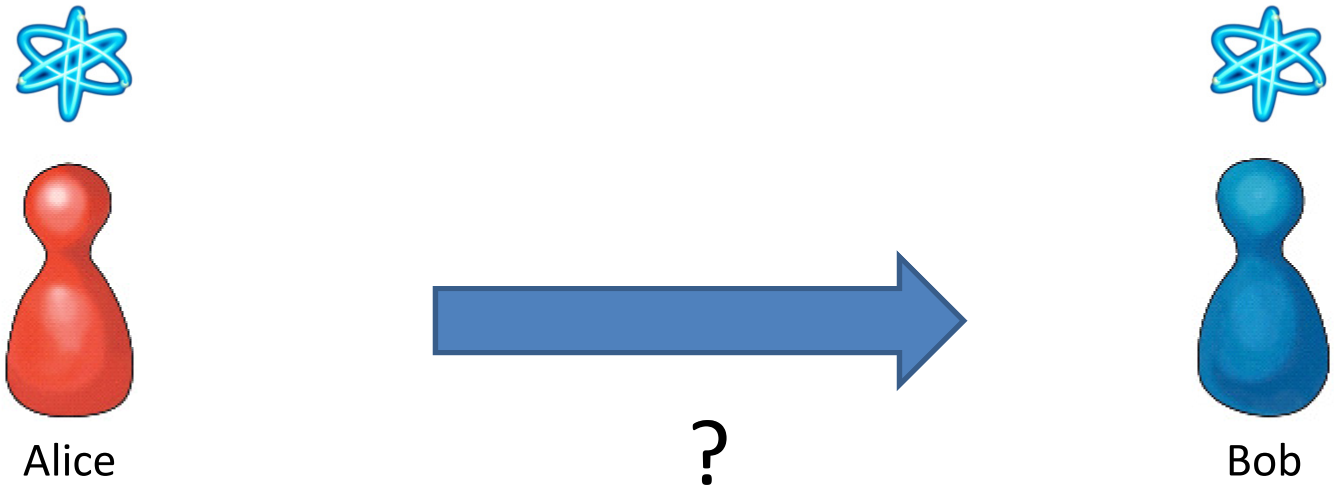


X: 00101100

$$X = Y\ ?$$

Y: 00101100

Deutsch-Josza Communication



X: 00101100

X = Y ?

Y: 00101100

Deutsch-Josza Communication

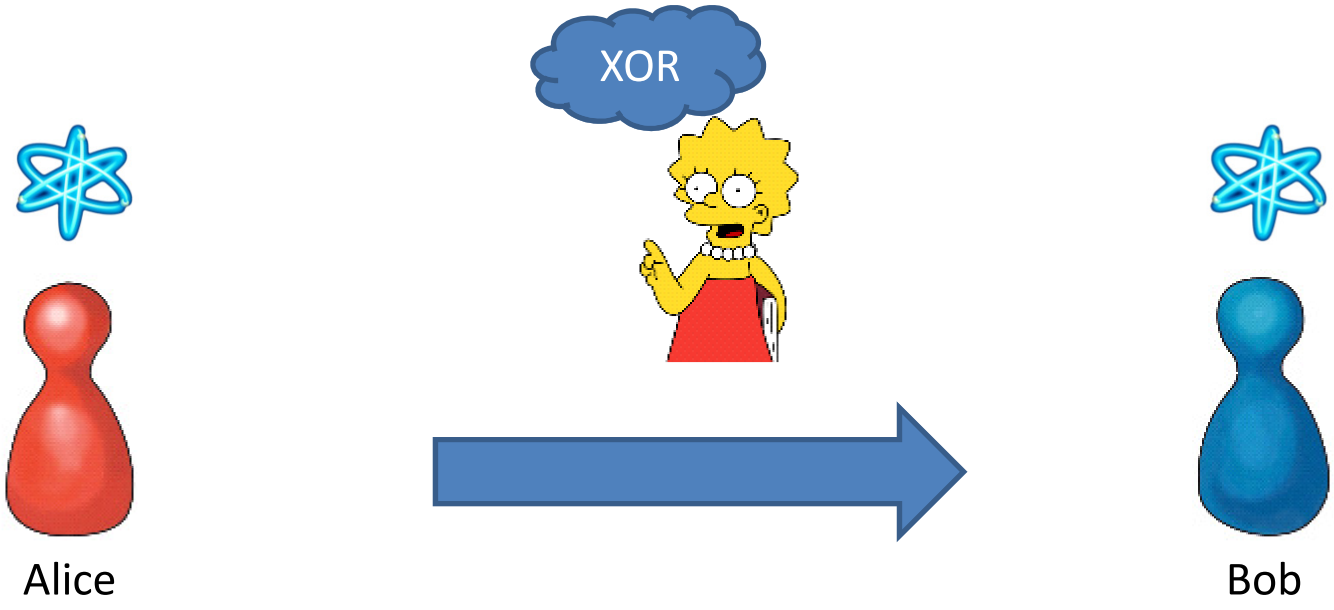
X: 00101100

X = Y ?

Y: 00101100

Promise: Hamming Distance = n/2 or 0

Deutsch-Josza Communication



X: 00101100

X = Y ?

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Promise: Hamming Distance = n/2 or 0

Deutsch-Josza Communication

X: 00101100

X = Y ?

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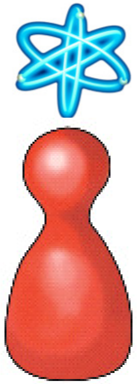
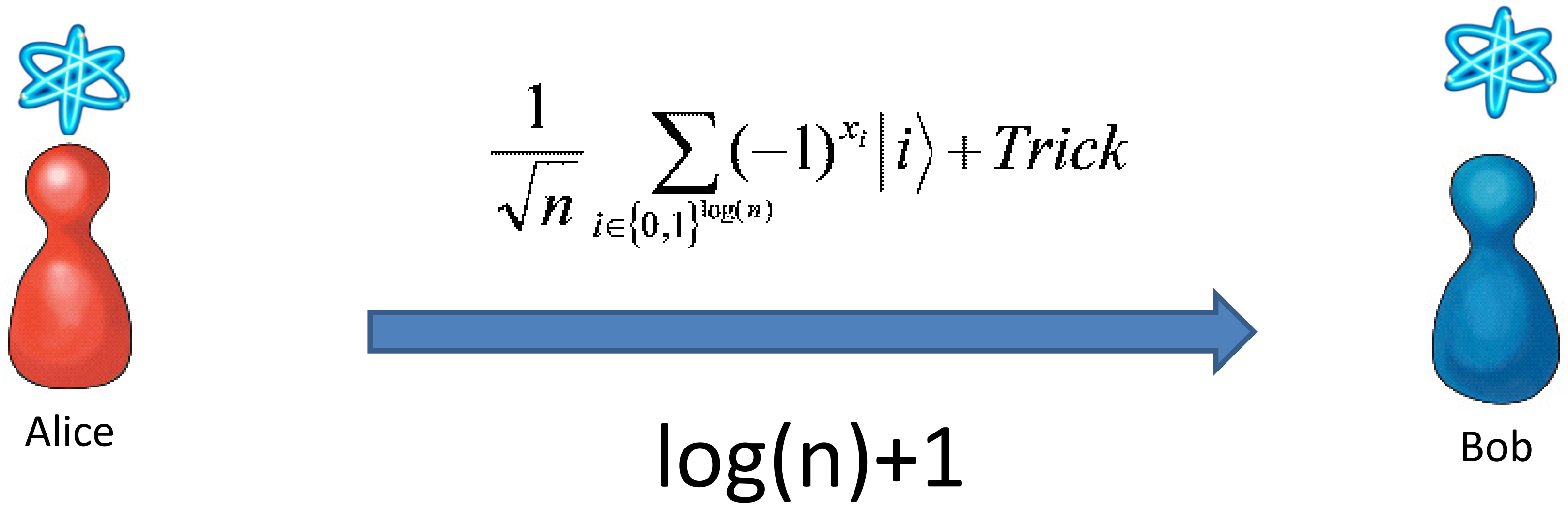
Deutsch-Josza Communication

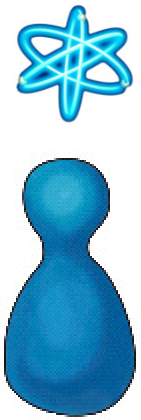


$$\frac{1}{\sqrt{n}} \sum_{i \in \{0,1\}^{\log(n)}} (-1)^{x_i} |i\rangle + Trick$$



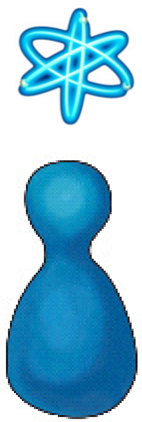
$\frac{1}{\sqrt{n}}\sum_{i\in\{0,1\}^{\log(n)}}(-1)^{x_i}|i\rangle + Trick$
Alice
log(n)+1
Bob

$$\frac{1}{\sqrt{n}} \sum_{i\in\{0,1\}^{\log(n)}} (-1)^{x_i} \left| i \right\rangle + Trick$$



Bob

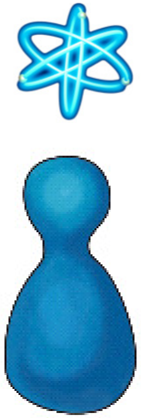
$$\frac{1}{\sqrt{n}} \sum_{i\in\{0,1\}^{\log(n)}} (-1)^{x_i} \left| i \right\rangle + Trick$$



Bob

$$\frac{1}{\sqrt{n}} \sum_{i\in\{0,1\}^{\log(n)}} (-1)^{x_i \oplus y_i} \left| i \right\rangle + Trick$$

$$\frac{1}{\sqrt{n}}\sum_{i\in\{0,1\}^{\log(n)}}(-1)^{x_i}\left|i\right\rangle + Trick$$



Bob

$$\frac{1}{\sqrt{n}}\sum_{i\in\{0,1\}^{\log(n)}}(-1)^{x_i\oplus y_i}\left|i\right\rangle + Trick$$

Deutsch-Jozsa Algorithm

What else is possible?

- Leader Election
- Disjointness
- [...]

Sources

- Caltech Introduction to Quantum Computing
  http://www.theory.caltech.edu/people/preskill/ph229/notes/chap1.pdf
- Distributed Quantum Computing
  Harry Buhrmann and Hein Röhrig
- Can quantum mechanics help distributed computing?
  Anne Broadbent and Alain Tapp
- Distributed Quantum Computing: A New Frontier in Distributed Systems or Science Fiction?
  Vasil S. Denchev and Gopal Pandurangan

Questions?