Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Self organizing robot gathering

Seminar in Distributed Computing

Christof Baumann
Mentor: Tobias Langner

ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

What is it all about

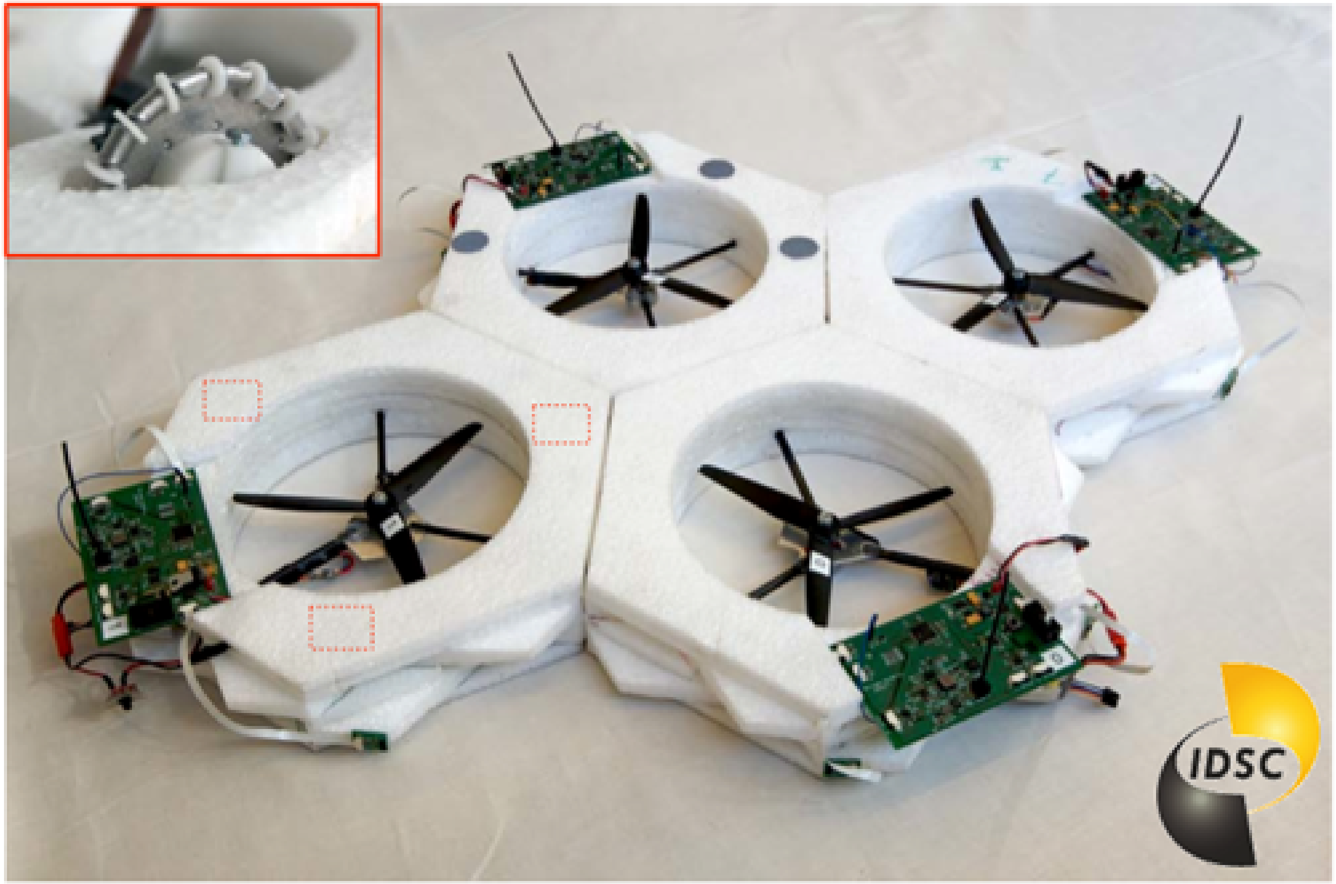
- n robots with restricted capabilities
- 2D plane setting
- They want to gather in a single point

Where gathering could be used

- Mars robots
  - Multiple robot types (to save money)
    - Robots equipped with radio
    - “Dumb” robots
  - Radio robots do jobs for the whole group
  - To exchange data they need to gather
- Military
  - Mine searching
  - Spy robots
- Task splitting
  - After gathering the main robot distributes the tasks
- Distributed Flight Array

ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Distributed Flight Array



Distributed Flight Array

- Movie

The paper

- Title: A Local $O(n^2)$ Gathering Algorithm
  - Bastian Degener
  - Barbara Kempkes
  - Friedhelm Meyer auf der Heide
  - (All at University of Paderborn in Germany)
- Published
  - at the Symposium on Parallelism in Algorithms and Architecture (SPAA)
  - in the year 2010

Overview

- Motivation
- Previous work
- The models
- The algorithm
- Conclusions
- Questions

Previous Work

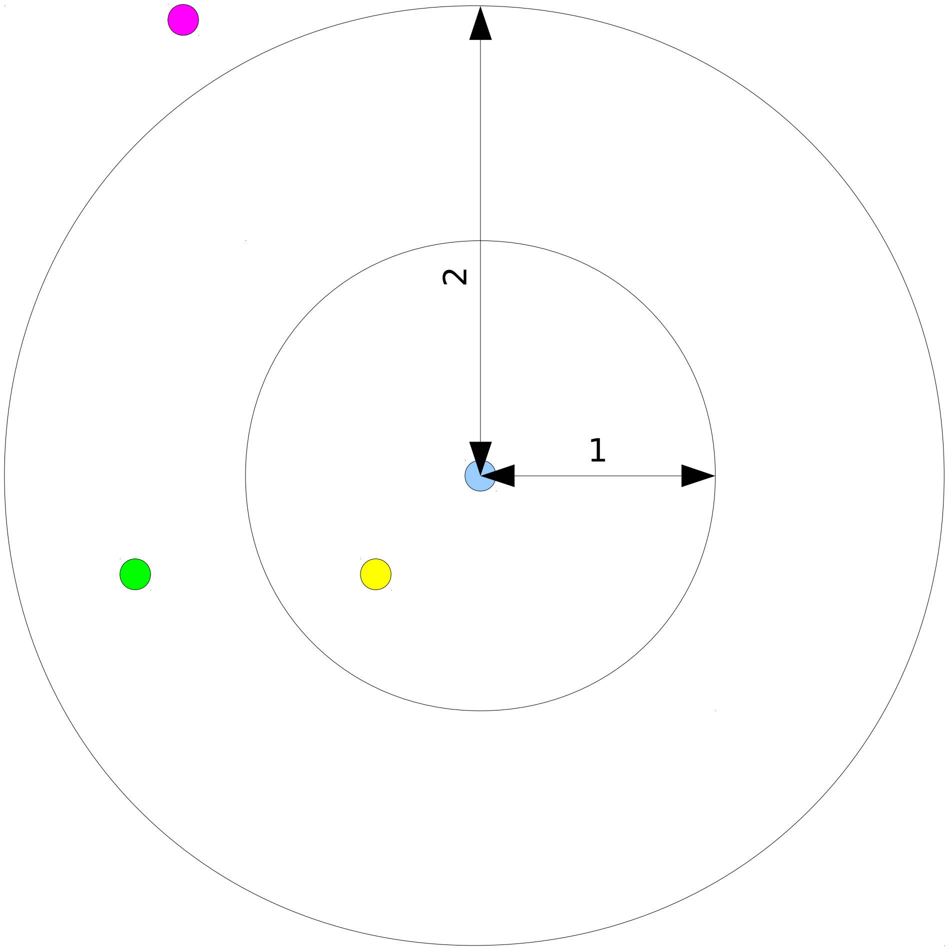
- No runtime bounds with a just local view
- All runtime bounds known rely on a global view
- Gathering if malicious robots are involved
- Robots that are not point sized but have an extent
  - View of robot can be blocked
- Compass model

Overview

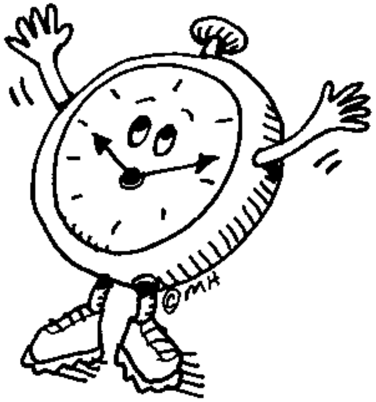
- Motivation
- Previous work
- The models
- The algorithm
- Conclusions
- Questions

Robot Model

- Limited viewing range
- Do not have a memory
- No common coordinate system
- Assign target positions to other robots within connection range
- Measure positions of other robots within viewing range



Time Model



- Just one robot active at the time
- Next robot chosen randomly
- Round model
  - Each robot is active at least once
- A round takes $O(n\log(n))$ steps in expectation
  - Coupon collector

ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Active vs. inactive Robots

- Active Robot
  - See positions of other robots
  - Tell robots target position
  - Move to own target position (max. distance of 2)
- Inactive Robot
  - Be told a target position
  - Move to the target told (max. distance of 3)

Overview

- Motivation
- Previous work
- The models
- The algorithm
- Conclusions
- Questions

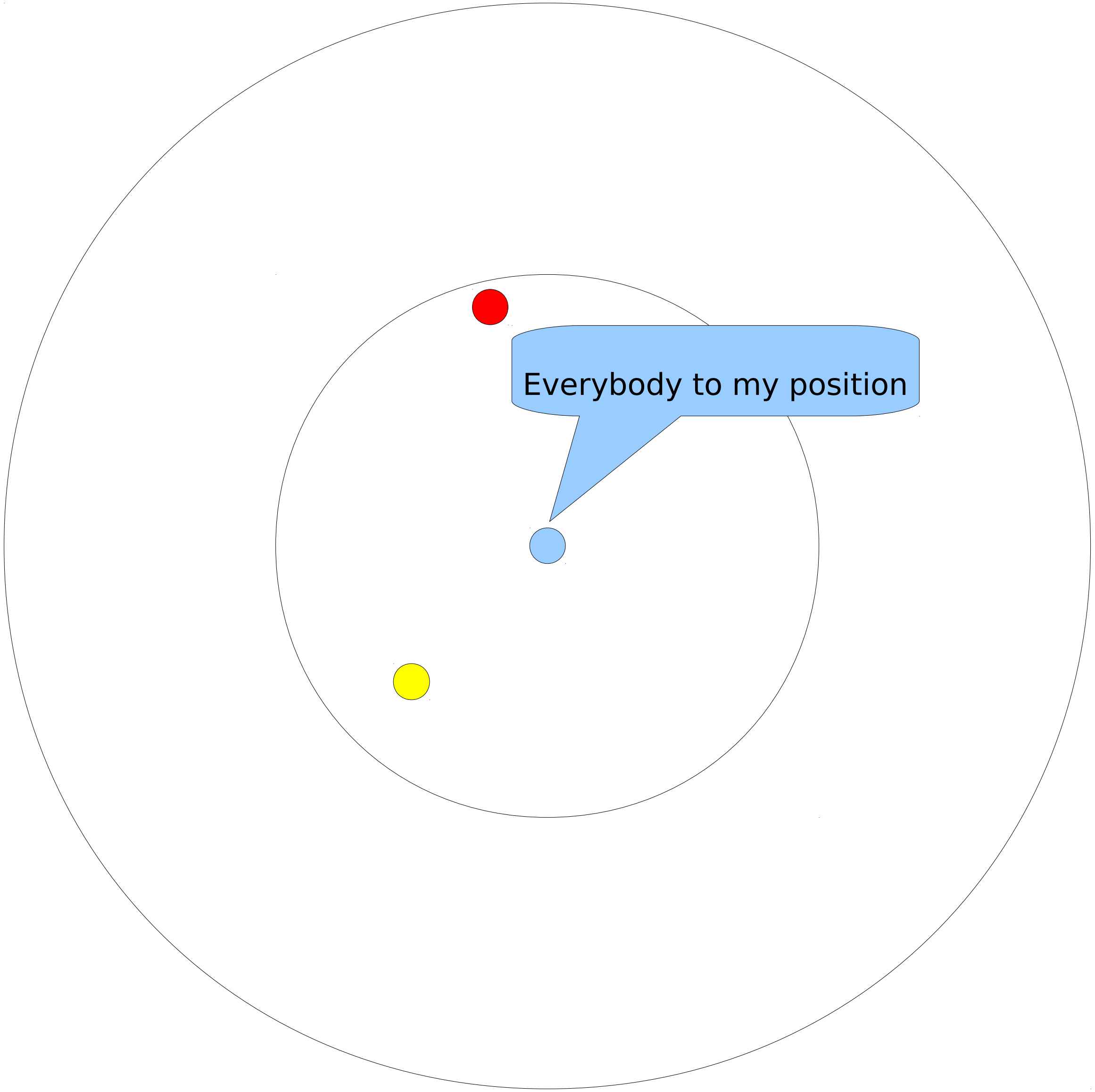
The algorithm

- The active robot executes one of
  - Termination
    - Just executed once
    - Complete the gathering
  - Fusion
    - Fuse two robots
    - Fused robots are treated as one
  - Reduction
    - Reduce the area of the convex hull of the network

ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

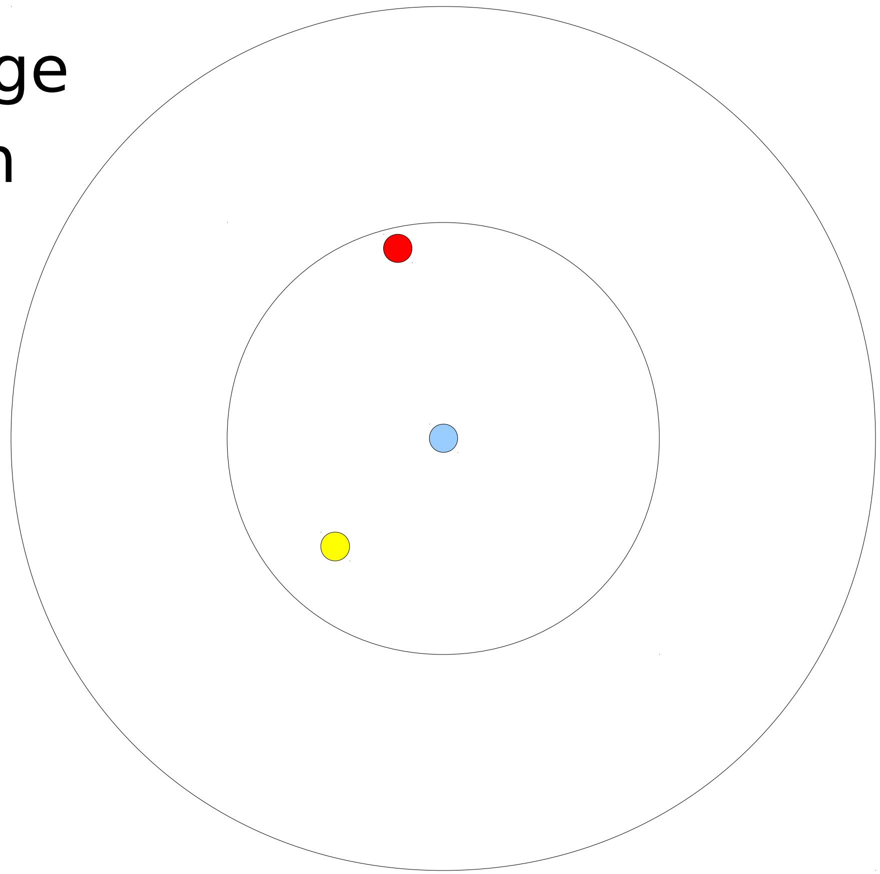
Termination

- If all robots are in connection range
- If this step is done we have gathered if the network was connected



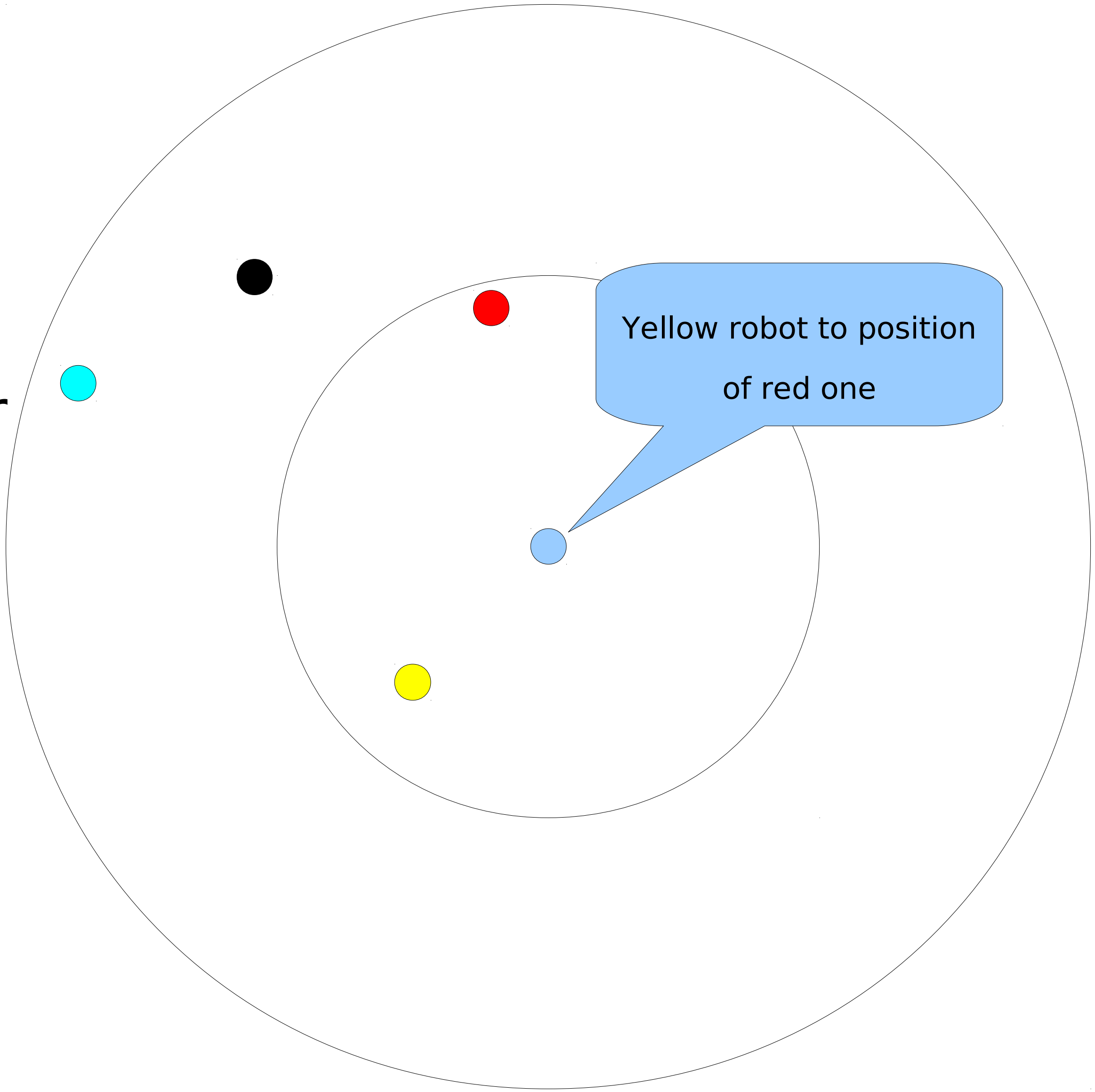
Network connectivity after termination step

- No robots in viewing range
- Robots just in connection range
- Nothing gets disconnected



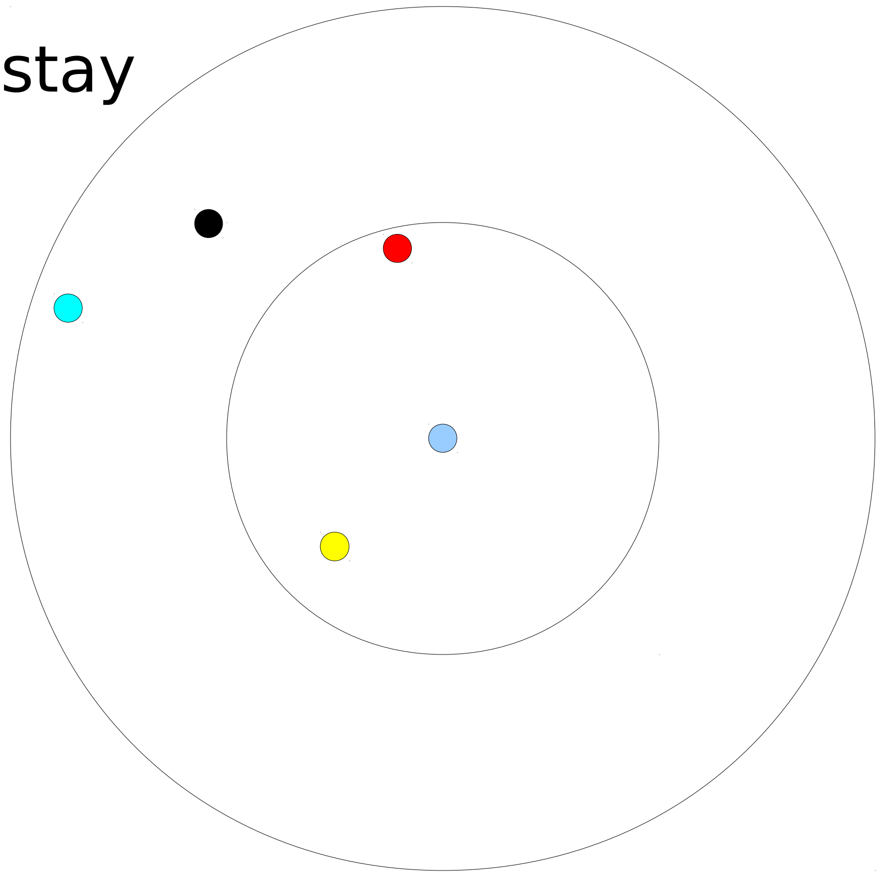
Fusion

- Fuse two (or more) robots together
- If there is a configuration in which these conditions hold
  - Robots still contained in the convex hull
  - Still connected



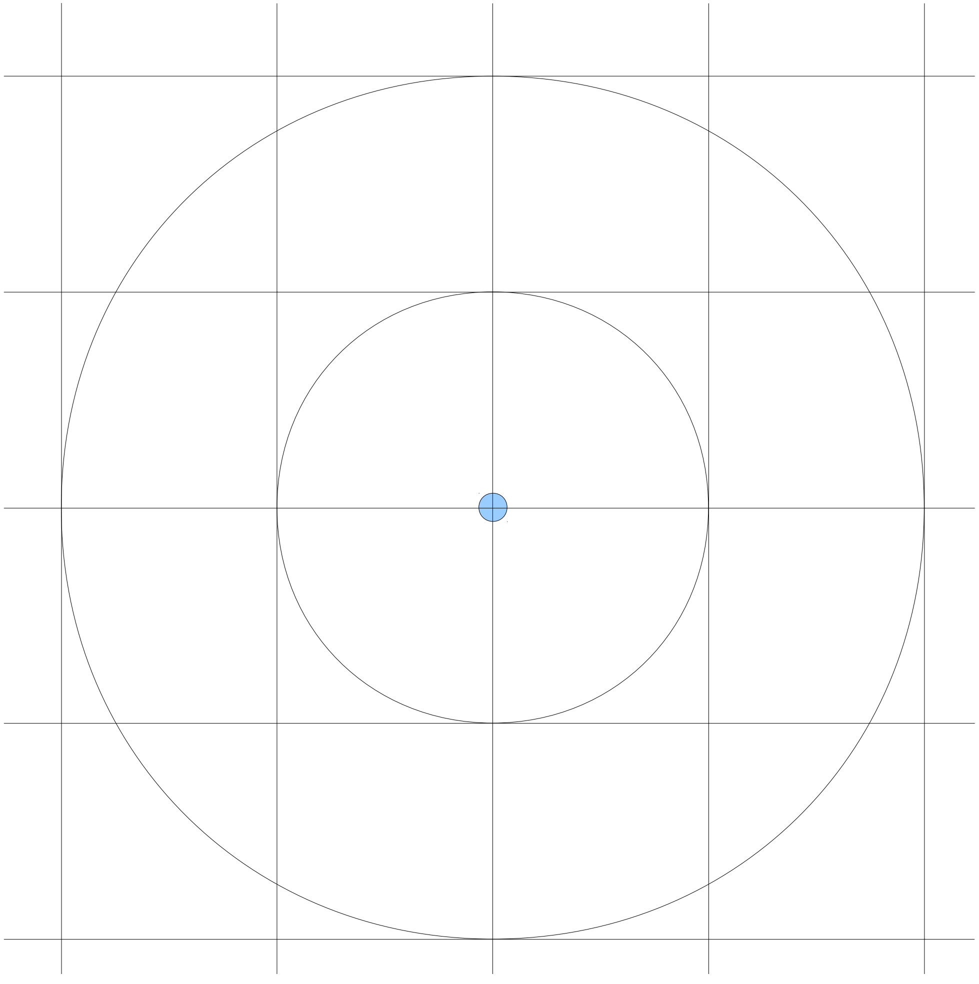
Network connectivity after fusion step

- Robots in viewing range stay connected by definition
- If it is not possible to fuse robots the third possibility is executed



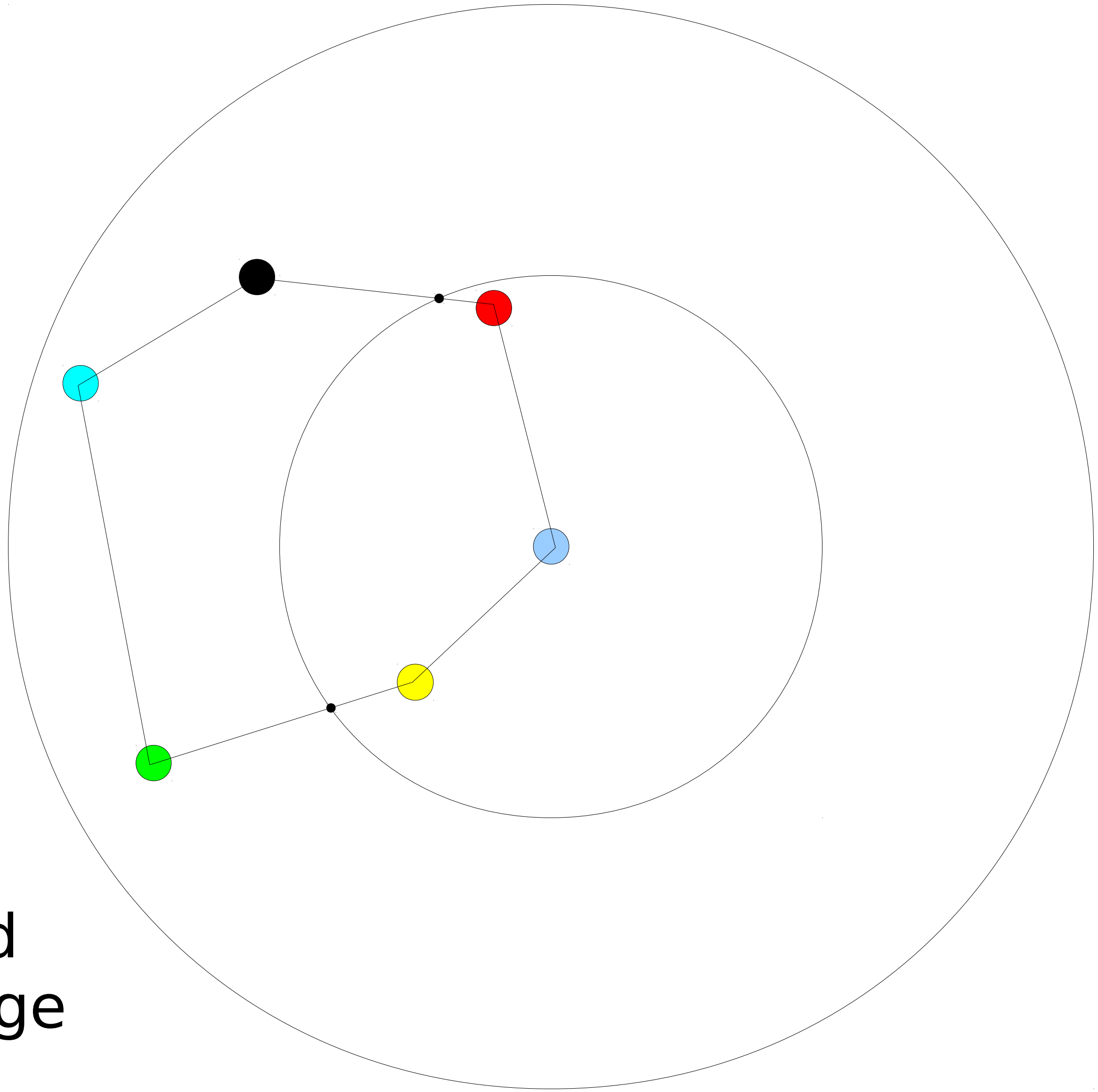
Lower bound for the # of robots in the connection range to have a fusion

- Define c as the number of nodes the active robot can see
- If c > 16 a fusion is possible
- Pigeonhole



Reduction

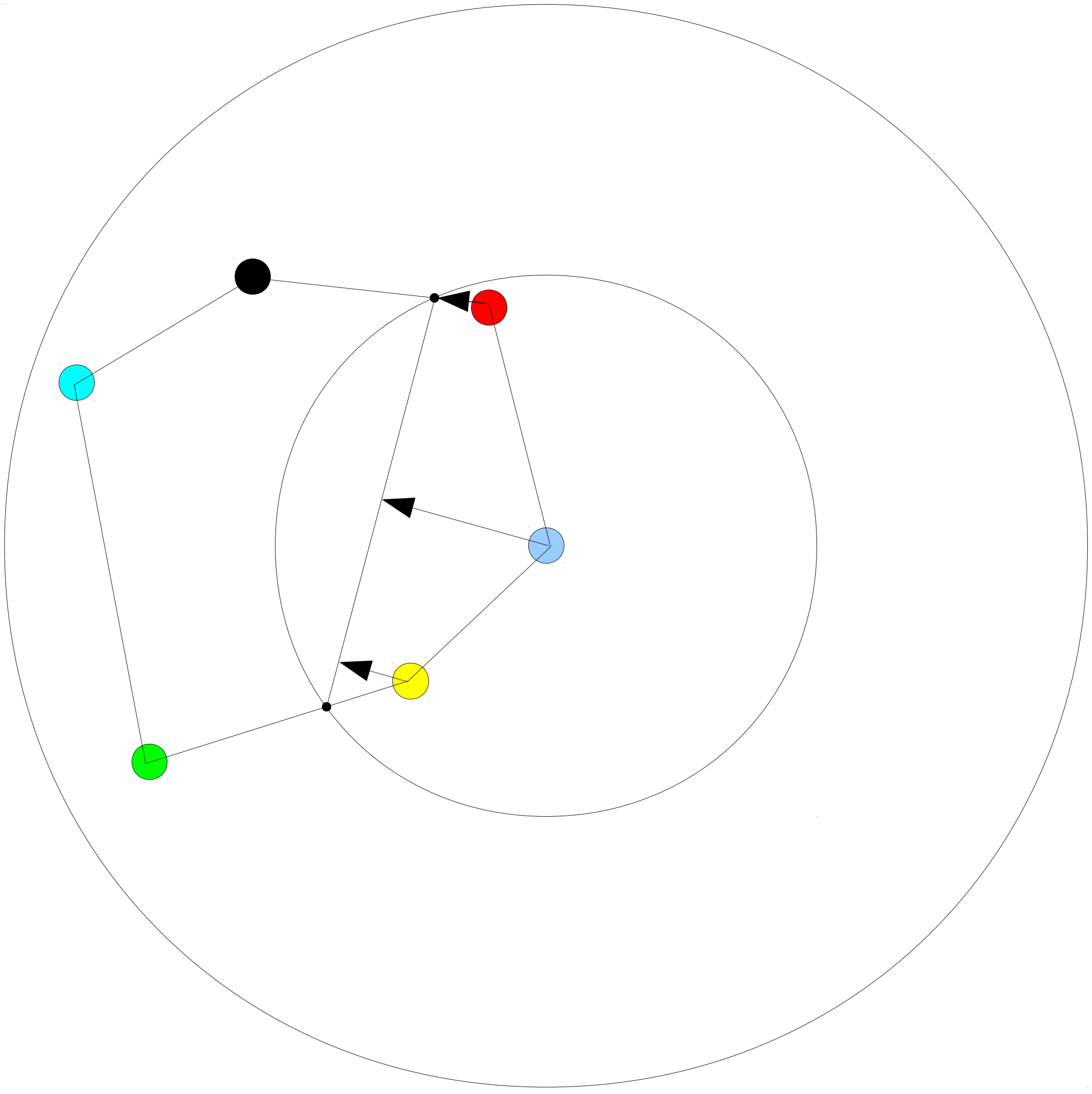
- If fusion not possible
- Compute the convex hull
- Compute intersections with maximum distance of convex hull and connection range



Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

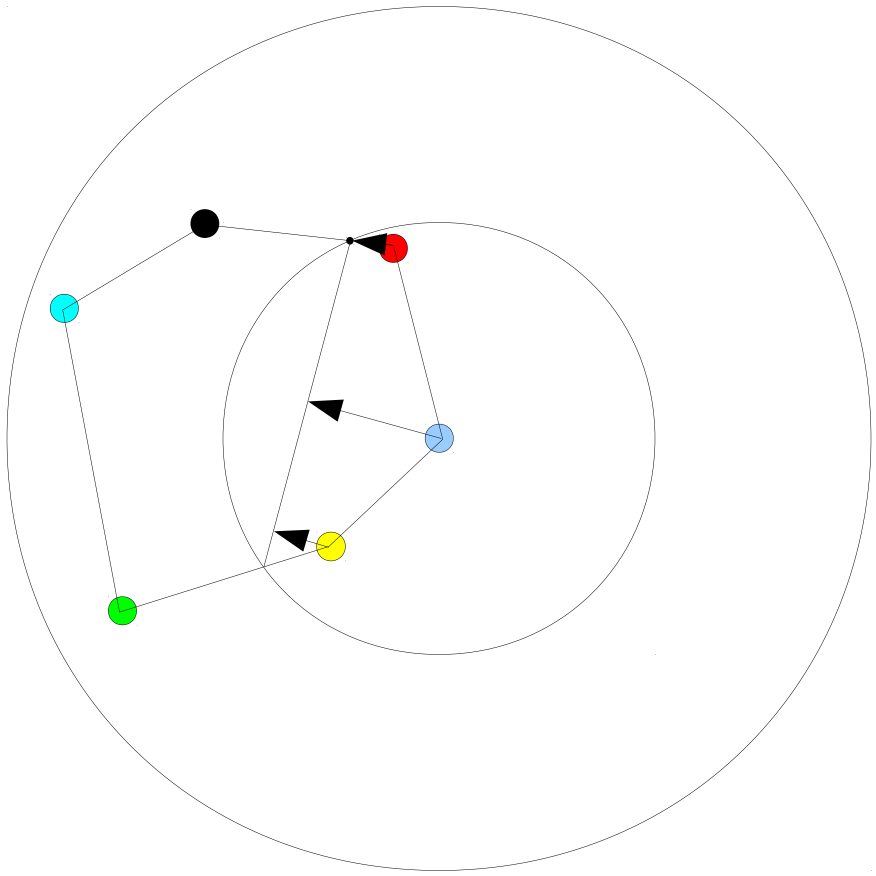
Reduction

- Compute line segment L between the points
- Move robots on the same side as the active robot to their closest point on L



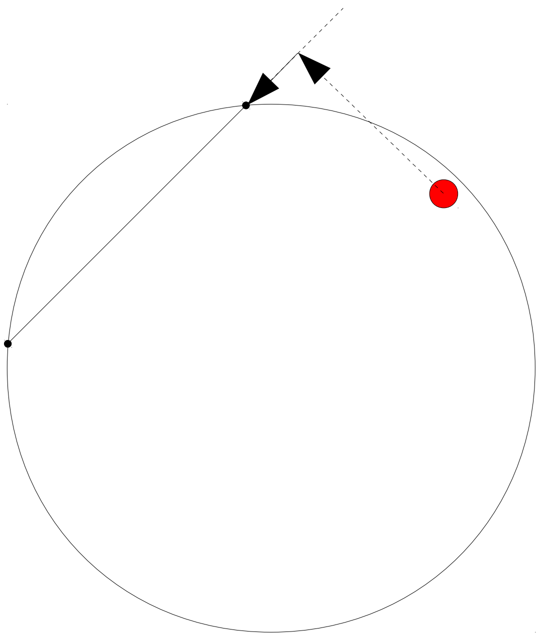
Network connectivity after reduction step (1/2)

- Only robots within the active robots connection range are moved
- Convex hull of active robot stays connected
- By projection the distance does not increase



Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Network connectivity after reduction step (2/2)



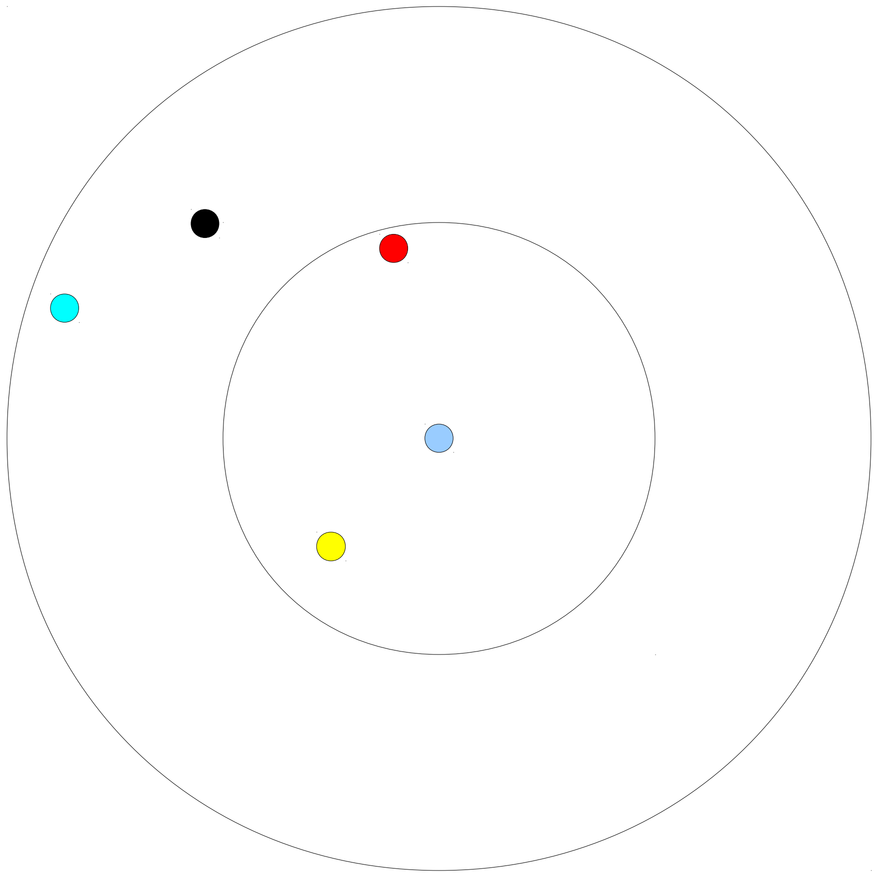
Run time Analysis

- In each round one of the 3 possibilities is executed
  - Termination
  - Fusion
  - Reduction
- If there's a bound for the maximum number of rounds for each of them we have a bound for the algorithm

Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

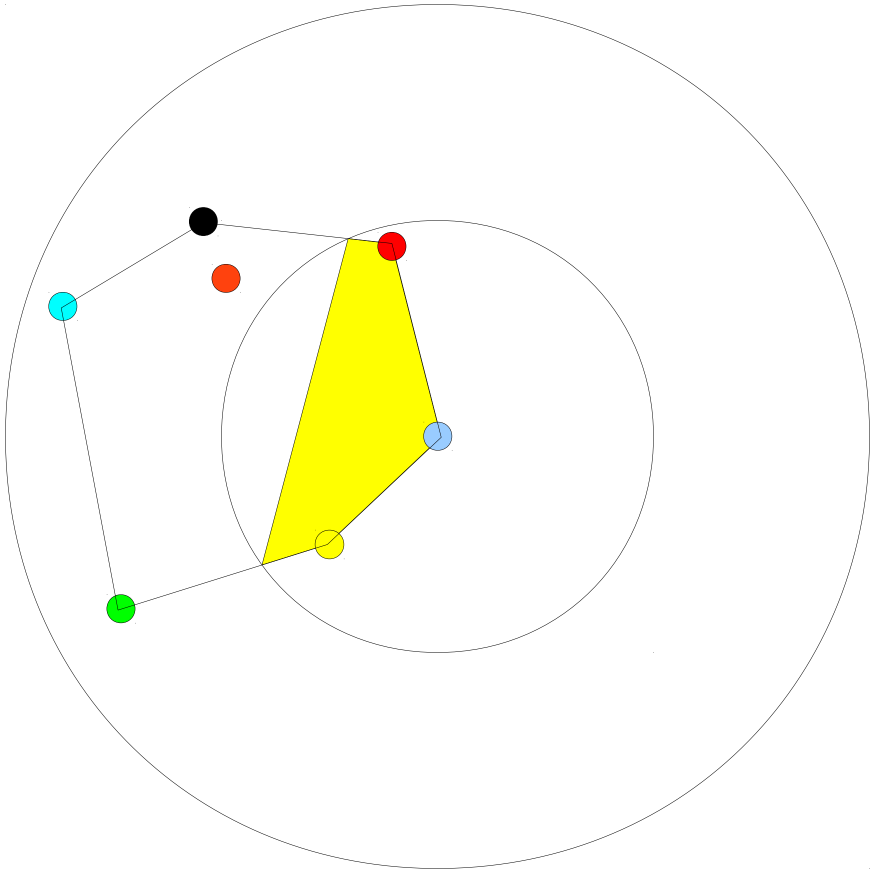
Progress Fusion

- Directly visible progress
- Easy to bound
- Maximally n-1 rounds with fusion
- Runtime: O(n)



Progress Reduction

- Reducing the size of the global convex hull
- We will prove that the area of the global convex hull is decreased in expectation by a constant factor in each round

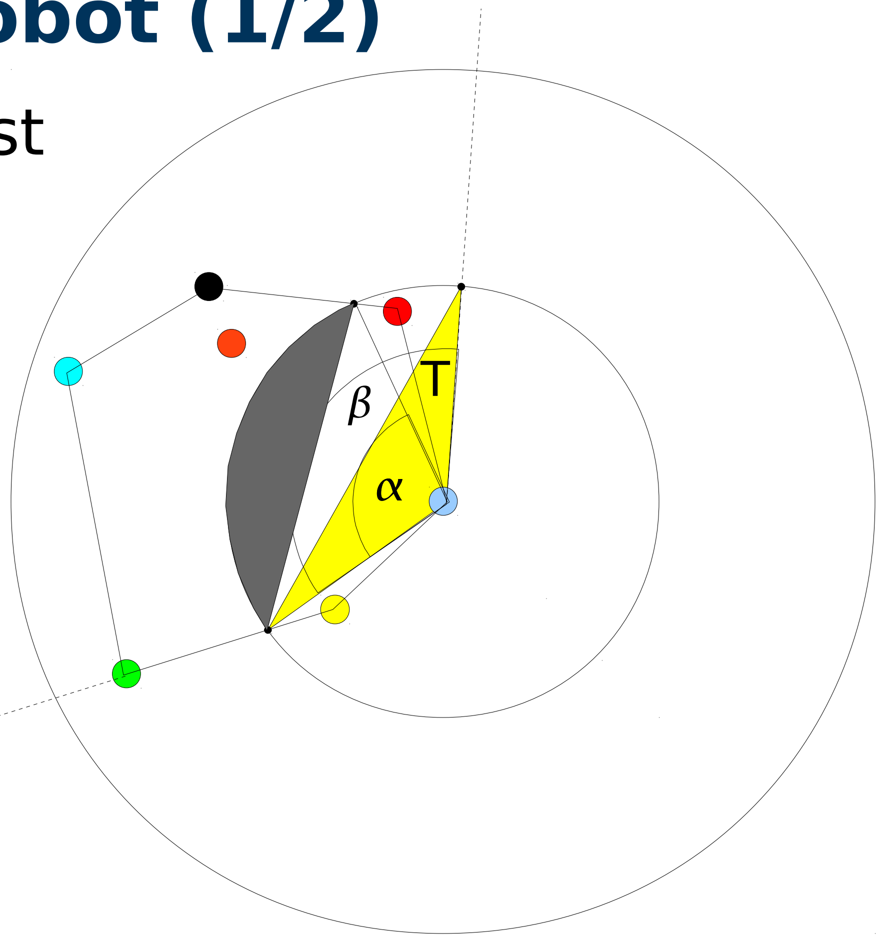


Bound the reduction area of a global convex hull vertex robot (1/2)

- The convex hull is at least reduced by the area of T

$$\sin\left(\frac{\beta}{2}\right)\cdot\cos\left(\frac{\beta}{2}\right)\geqslant\sin\left(\frac{\alpha}{2}\right)\cdot\cos\left(\frac{\beta}{2}\right)$$

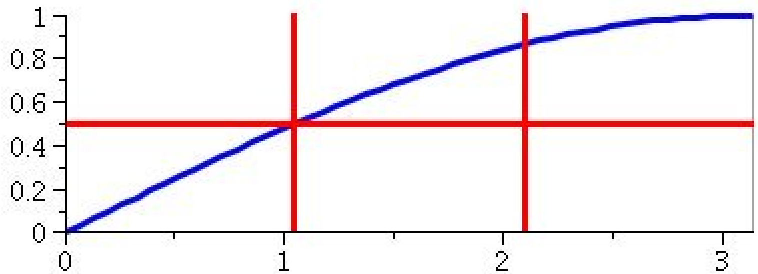
- Because of $\beta\geqslant\alpha$
  - The global convex hull contains the viewing range of the active robot at the beginning of a time step



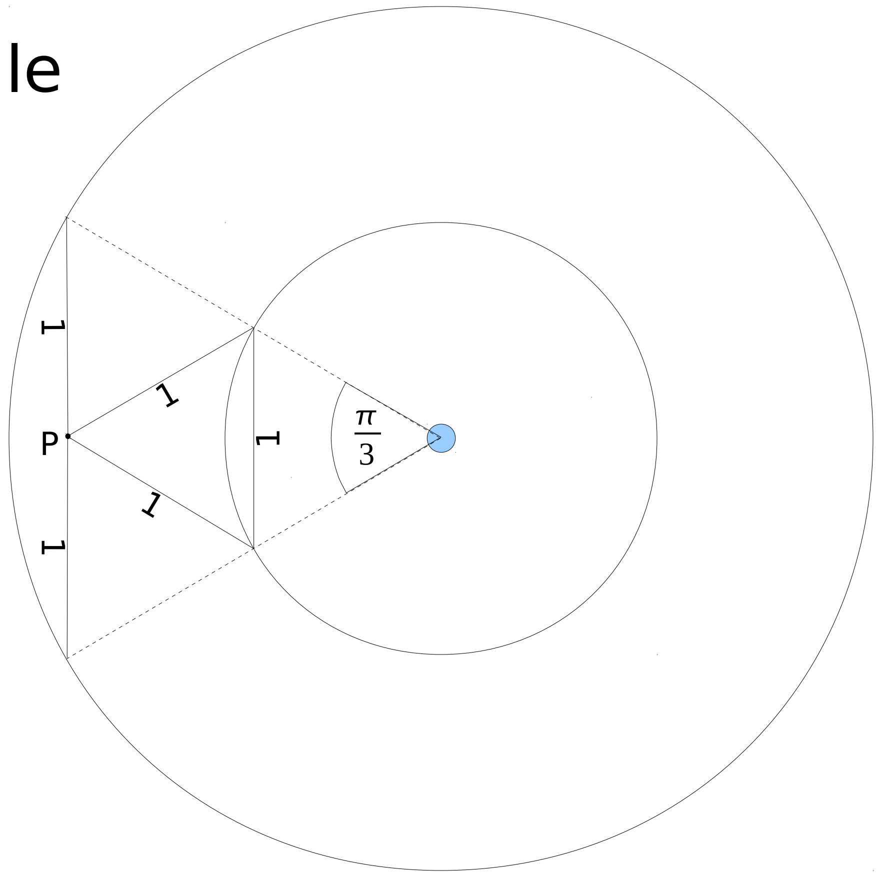
Bound the reduction area of a global convex hull vertex robot (2/2)

- Give a bound for the angle seen by the active robot

$$\alpha \geqslant \frac{\pi}{3}$$

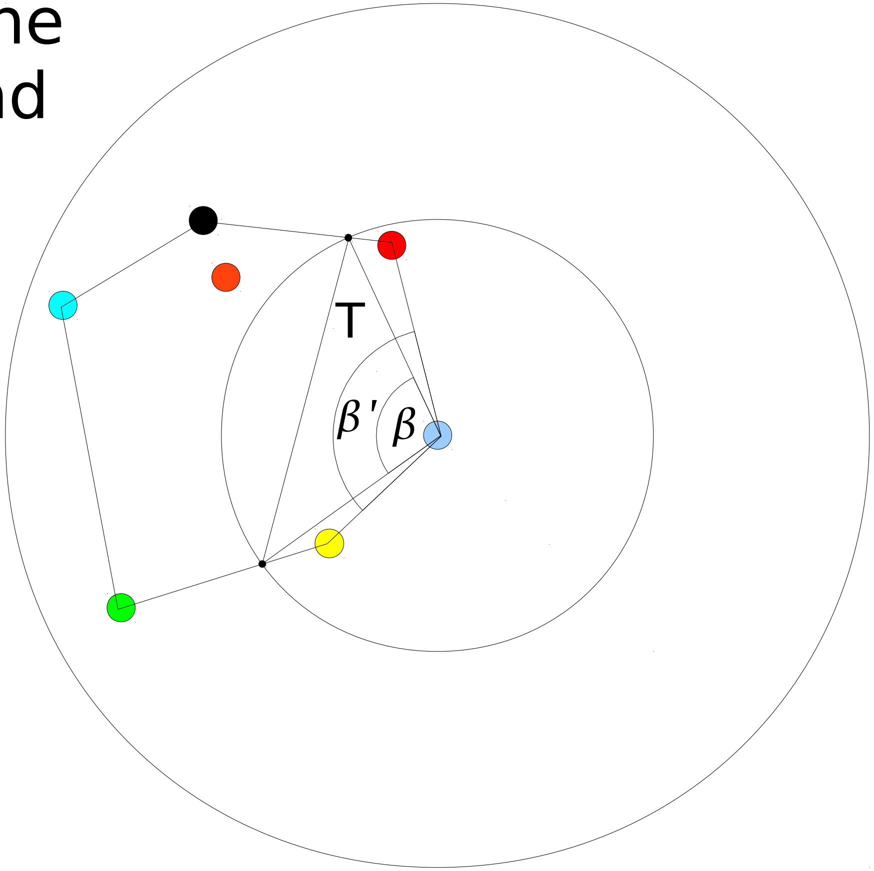


$$\sin\left(\frac{\alpha}{2}\right)\cdot\cos\left(\frac{\beta}{2}\right) \geqslant \frac{1}{2}\cdot\cos\left(\frac{\beta}{2}\right)$$



Bound the reduction area of a round

- We want to use the sum of internal angles of the global convex hull to get the area truncated in one round
  - $\sum \beta_i' = \pi \cdot (m-2)$
- If a robot that is a vertex of the convex hull is the first one active in its neighborhood then $\beta' \geq \beta$ holds



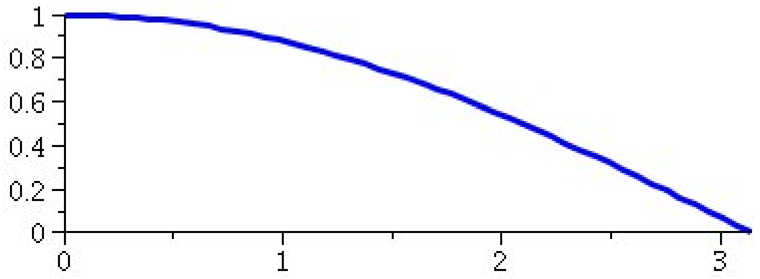
Bound the expected reduction area of a single step (1/2)

- The expected truncated area by the active vertex robot is

$$E[a] \geqslant Pr[\text{robot is the first activated in connection range}] \cdot (\text{area truncated})$$

$$= Pr[\text{robot is the first activated in connection range}] \cdot \frac{1}{2}\cos\left(\frac{\beta}{2}\right)$$

$$\geqslant Pr[\text{robot is the first activated in connection range}] \cdot \frac{1}{2}\cos\left(\frac{\beta'}{2}\right)$$



Bound the expected reduction area of a single step (2/2)

- Probability that a vertex robot with c neighbors is not moved before its activation

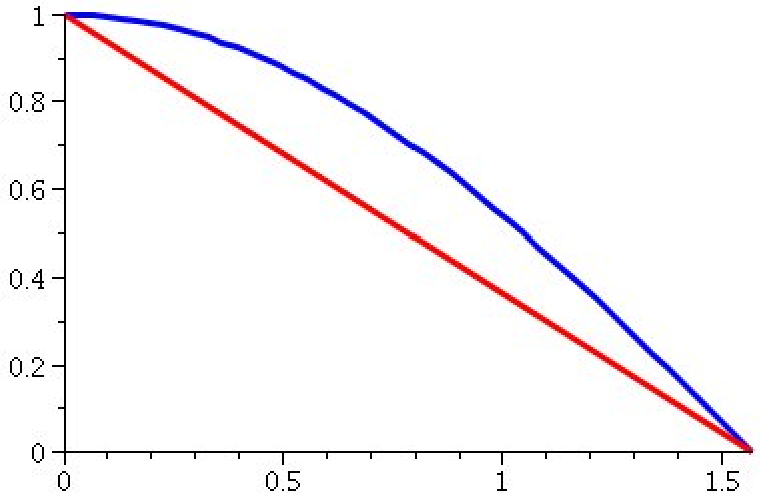
$$\sum_{t=0}^{\infty} \frac{1}{n}(1 - \frac{c}{n})^t = \frac{1}{n} \cdot \frac{1}{1-(1-\frac{c}{n})} = \frac{1}{n} \cdot \frac{1}{\frac{c}{n}} = \frac{1}{c}$$

- c is the maximum number of robots in viewing range without a fusion
- We already know that c<16

$$E[a] \geqslant \frac{1}{c} \cdot \frac{1}{2} \cos\left(\frac{\beta'}{2}\right)$$

Bound the reduction area in a round

- Sum up



Runtime of the algorithm

- Fusions
  - maximally n-1
- Reductions
  - In the beginning the convex hull has maximum area of n^2
  - We have a constant reduction in each round
  - We need $O(n^2)$ rounds in expectation
    - Expectation comes from the stochastic round model
    - The algorithm itself is deterministic

Overview

- Motivation
- Previous work
- The models
- The algorithm
- Conclusions
- Questions

Conclusions / Critics

- The constraint that the active robots can give orders is very strong
- The randomized round model is hard to implement in practice
- Just one active robot at the time

ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Overview

- Motivation
- Previous work
- The models
- The algorithm
- Conclusions
- Questions