Principles of Distributed Computing
Exercise 4: Sample Solution

1 Shared Sum

In the following, let X (initialized to 0) always denote the shared register used to hold the sum $x = \sum_{i=1}^{n} x_i$, and assume that all x_i (and thus also x) are initially 0. Denote by Δx_i the amount by which x_i is changed by process p_i at some time, i.e., if $x_i := x_i'$ is assigned by p_i, then $\Delta x_i = x_i' - x_i$.

a) To update x, p_i calls fetch-and-add$(X, \Delta x_i)$. Therefore, X changes exactly the same as x_i and holds the correct value. Since no process has to wait or retry, we have neither lockouts nor deadlocks. A simple read on X (or fetch-and-add$(X, 0)$) gets the current value of x.

b) An update is done by the following code:

```
1: x := X
2: while not compare-and-swap(X, x, x + Δxi) do
3:     x := X
4: end while
```

The loop is left after X changed by Δx_i exactly once, thus the code is correct. Again, x can be obtained by a simple read. Since the compare-and-swap may only fail if another process p_j changed the value of X between p_i reading it and calling compare-and-swap, there is no deadlock. However, other updates may delay a change by some p_i indefinitely, hence lockouts are possible.

c) A write is implemented by

```
1: x := load-link(X)
2: while not store-conditional(X, x + Δxi) do
3:     x := load-link(X)
4: end while
```

and is correct for the same reasons as in **b)**. Reads are again simple. However, the solution differs from **b)** in that we may have deadlocks, since e.g. two processes can fight endlessly for getting the register linked to them for sufficiently long to write a value (At least for the case where a load-link can destroy the link to the register of another processor; i.e. weak LL/SC).

d) It can be done. We use a special encoding on X. Either it stores a regular value and $\perp$ (i.e., $(x, \perp)$) or the value and an additional identifier identifier $id(i)$ of a process p_i. A node will effectively acquire a lock on X by writing its ID to X and only afterwards write its update to X.

When x_i is changed, p_i executes

1: **while true do**
2:     $(x, id) := X$ // simple read
3:     $(x, id) :=$ compare-and-swap$(X, (x, \perp), (x, id(i)))$ // try to lock X with own ID
4:     **if** $id = id(i)$ **then**
5:         $X := (x + \Delta x_i, \perp)$ // regular write, but compare-and-swap would also do
6:         **break while loop**
7:     **end if**
8: **end while**

Because writing by compare-and-swap works only if the second argument equals the value of the register, once a process "locks" X with its identifier, no other process may do so until the same process performs the write enclosed in the if-condition. Thus, this write happens exactly if the compare-and-swap was successful. The only reason to check the identifier by an if-statement rather than using compare-and-swap again is that we need to ensure that the process leaves the loop after changing X by Δx_i. On the other hand, the while loop can only be left after a succesful write, thus X is updated correctly. Reads are again plain reads.

As before, the solution is free of deadlocks: At least one process can write, because after each write the ID part of X contains $\perp$, i.e., one process will succeed in "locking" X. As in **b)** and **c)**, the solution is prone to lockouts.