



## Computer Engineering II

### Solution to Exercise Sheet 12

#### 1 Quiz Questions

- a) No: The (supposed) security depends on not knowing the shift  $x$ . If CAESAR is applied twice, you just chose another shift (and in the worst case, cancel out the encryption).
- b) No. E.g.,  $2 * 3 * 5 - 1 = 29$ , which is prime. But  $30 * 29 - 1 = 869 = 11 * 79$ . Even the first part is not correct, e.g.:  $2 * 3 * 5 * 7 = 210$  and  $210 - 1 = 11 * 19$ .
- c) Yes. An attacker could just flip the bit of the message.
- d) No. The attacker could just hash the modified message as well.

#### 2 Secret Sharing

- a) example execution: Let  $a_1 = 3$  and  $s = 2$ , with 2 neighbors. Thus,  $f(x) = 2 + 3x$ . We distribute, e.g., (2, 8) and (3, 11). With both pairs,  $s = 2$  can be recovered.
- b) Without obtaining  $t$  pairs,  $k$  can take any value, i.e.,  $t - 1$  pairs reveal no information on  $k$ .

#### 3 The One Time Pad

- a) If you apply the same one time pad twice, it cancels out, leaving you with the original message.
- b) Essentially, you created a new one time pad. If both are truly random, then this method is not more secure, but also not less, it is the same.
- c) The beauty of the one time pad is that it transforms the message into a random message. As thus, any string of length  $k$  could be the original message - you still know nothing except for the length of the message.
- d) Let  $x$  be the OTP.  $c_1 \oplus c_2 = m_1 \oplus x \oplus m_1 \oplus x = m_1 \oplus m_2$ , i.e., the one time pad cancels out. You don't have it decrypted yet, but it is a lot more information than just a random string.
- e) You can get, e.g.,  $m_3 \oplus m_4$ , using similar techniques as above. I.e.,  $c_3 \oplus c_2 = m_3 \oplus x$  and  $c_4 \oplus c_3 = m_4 \oplus x$ , leading to  $c_4 \oplus c_2 = m_4 \oplus m_3$ .

## 4 Diffie-Hellman Key Exchange

- a) The primitive roots are 3 and 5.
- b) Alice sends  $3^4 = 81 = 4 \pmod{7}$  and Bob sends  $3^2 = 9 = 2 \pmod{7}$ . As thus, they agree on  $4^2 = 16 = 2 \pmod{7}$  (or  $2^4 = 16 = 2 \pmod{7}$ ).
- c) (individual solutions)
- d) Alice picked  $k_A = 3$ , Bob picks  $k_B = 2$ . Alice sends  $3^3 = 27 = 2 \pmod{5}$  to Bob and Bob sends  $3^2 = 9 = 4 \pmod{5}$  to Alice. As thus, they agree on  $2^2 = 4 \pmod{5}$  (or  $4^3 = 64 = 4 \pmod{5}$ ).

## 5 Message Authentication

- a) E.g., use sequence numbers.
- b) The answer is no to both: Take any  $m$  and  $m' = m + p$ , then  $h(m) = h(m')$ . Similarly, given any  $1 \leq m \leq p - 1$ ,  $h(m) = m$ .
- c) Use a large prime  $p$  with a primitive root  $g$ . With  $m$  being the message, let the hash be  $h(m) = g^m \pmod{p}$ . Now, finding an  $x$  s.t.  $h(x) = h(m)$  is the desired hash is equivalent to solving the discrete logarithm problem.