Discrete Event Systems
Exercise 8

1   Night Watch

In order to improve their financial situation, Roland and Stefan also work at nights. Their task is to guard a famous Swiss bank which, from a architectonic perspective, looks as follows:
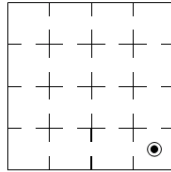

Figure 1: Offices of a Swiss bank.

Thus, there are 4x4 rooms, all connected by doors as indicated in the figure.

In a first scenario, Roland and Stefan always stay together. They start in the room on the upper left. Every minute, they change to the next room, which is chosen uniformly at random from all possible (adjacent) rooms.

   a) Compute the probability (in the steady state) that Roland and Stefan are in the room where the thief enters the bank (indicated with ⊙)!

   b) Since Roland and Stefan are very strong, they can easily catch a thief on their own. Thus, in a second scenario, they decide that it be smarter to patrol individually: After every minute, each of them chooses the next room *independently*. What is now the probability that at least one of them is in the room where the thief enters?

Note: This exercise is inspired by an exam question of winter term 2004/5.

2   Probability of Arrival

In the script, there is a lemma saying that the probability of arrival can be computed as

$$f_{ij} = p_{ij} + \sum_{k:k\neq j} p_{ik} f_{kj}.$$

Prove this lemma.

3   "Hopp FCB!"

Besides its moodiness, the *FC Basel* (FCB) soccer club is confronted with yet another problem: sporadically, its players get sick. Assume that the whole team consists of n players. Assume that the time period a fit player remains fit is exponentially distributed with parameter μ (independently of the state of the other players). On the other hand, the time a sick player remains sick is exponentially distributed with parameter λ.

a) Model the situation as a birth-and-death Markov process where the states denote the number of players which are fit.

b) Derive a formula for the probability that exactly i players are fit.

c) Assume that the FCB has 20 players, and that $\lambda^{-1} = 4$ months and $\mu^{-1} = 10$ months. Calculate the probability that the FCB can participate at a given match.