ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Distributed Computing Group

HS 2008 Prof. Dr. Roger Wattenhofer, Nicolas Burri, Philipp Sommer

Ad Hoc And Sensor Networks
Exercise 7

Assigned: November 3, 2008
Due: November 10, 2008

1 Slotted Aloha

In this exercise we want to analyze 'Slotted Aloha' for the case that the number of stations n is not exactly known. We assume that in each time slot each station transmits with probability p. In the lecture you saw that the probability that the slot can be used (i.e. the probability that exactly one station transmits) is

$$\Pr(\text{success}) = n \cdot p(1-p)^{n-1}.$$

If n is fixed, we can maximize the above expression and get the optimal p as shown in the lecture. Now assume that the only thing we know about n is $A \leq n \leq B$.

a) Which value p maximizes $\Pr(\text{success})$ for the worst $n \in [A, B]$?

b) What is this 'worst case optimal' value for p if $A = 100$ and $B = 200$?

2 Walsh Codes

In the lecture you have learned about Walsh codes and how they are recursively constructed (see page 34 of the notes of chapeter 7). In this exercise you will prove two fundamental properties of those codes.

2.1 Orthogonality

Prove that the code words of a Walsh code are pairwise orthogonal (as mentioned in the lecture). As an example, consider the Walsh code $\mathcal{C}_2$ of length 4. We have

$$\mathcal{C}_2 = \{(+1,+1,+1,+1), (+1,+1,-1,-1), (+1,-1,+1,-1), (+1,-1,-1,+1)\}.$$

For all 6 possible pairs of code words, we can easily verify that their inner product is 0.

2.2 Balance of the Code Words

From the recursive construction of Walsh codes, it is obvious that the word of all ones is always a code word ($(+1, +1, \ldots, +1) \in \mathcal{C}$). Prove that for all other code words, half of the components are $+1$ and half of the components are -1, i.e. prove that the code words of a Walsh code are balanced.