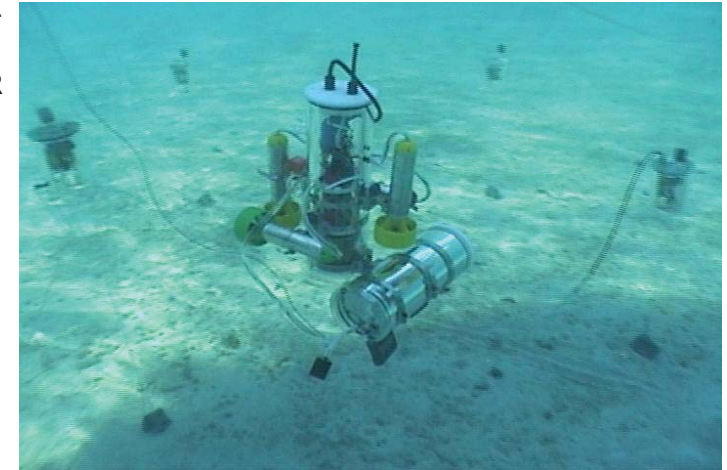


# Capacity

## Chapter 11

## Underwater Sensor Networks

- Static sensor nodes plus mobile robots
- Dually networked
  - optical point-to-point transmission at 300kb/s
  - acoustical broadcast communication at 300b/s, over hundreds of meters range.
- Project AMOUR [MIT, CSIRO]
- Experiments
  - ocean
  - rivers
  - lakes



## Rating

- Area maturity

First steps

Text book

- Practical importance

No apps

Mission critical

- Theory appeal

Booooooring

Exciting

## Overview

- Capacity and Related Issues
- Protocol vs. Physical Models
- Capacity in Random Network Topologies
- Achievable Rate of Sensor Networks
- Scheduling Arbitrary Networks

## Fundamental Questions

- How much communication can you have in a wireless network?
- How long does it take to meet a given communication demand?
- How much spatial reuse is possible?
- What is the **capacity** of a wireless network?
  
- Many **modeling issues** are connected with these questions.
- You can ask these questions in many **different ways** that all make perfect sense, but give different answers.
  
- In the following, we look at a few results in this context, unfortunately only superficially.



## Motivation

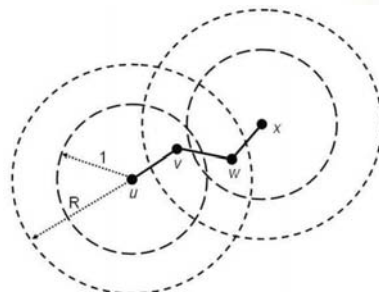
- Spatial capacity is an indicator of the “data intensity” in a transmission medium.
  
- The capacity of some well-known wireless technologies
  - IEEE 802.11b 1,000 bit/s/m<sup>2</sup>
  - Bluetooth 30,000 bit/s/m<sup>2</sup>
  - IEEE 802.11a 83,000 bit/s/m<sup>2</sup>
  - Ultra-wideband 1,000,000 bit/s/m<sup>2</sup>
  
- The wireless capacity is a function of the physical layer characteristics such as available **bandwidth or frequency**, but also how well the **protocols** on top of the physical layer are implemented, in particular media access. As such capacity is a theoretical framework for MAC protocols.



## Protocol Model

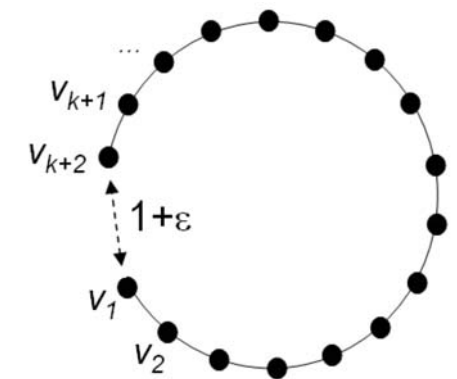
- For lower layer protocols, a model needs to be specific about interference. A **simplest interference model** is an extension of the UDG. In the protocol model, a transmission by a node in at most distance 1 is received iff there is no conflicting transmission by a node in distance at most R, with  $R \geq 1$ , sometimes just  $R = 2$ .

- + Easy to explain
- Inherits all major drawbacks from the UDG model
- Does not easily allow for designing distributed algorithms/protocols
- Lots of interfering transmissions just outside the interference radius R do not sum up
- Can be extended with the same extensions as UDG, e.g. QUDG



## Hop Interference (HI)

- An often-used interference model is hop-interference. Here a UDG is given. Two nodes can communicate directly iff they are adjacent, and if there is no concurrent sender in the  $k$ -hop neighborhood of the receiver (in the UDG). Sometimes  $k = 2$ .
  
- Special case of the protocol model, inheriting all its drawbacks
- + Simple
- + Allows for distributed algorithms
- A node can be close but not produce any interference (see picture)
  
- Can be extended with the same extensions as UDG, e.g. QUDG





## Measures for Network Capacity

- **Throughput capacity**
  - Number of successful packets delivered per time
  - Dependent on the traffic pattern
  - *E.g.: What is the maximum achievable, over all protocols, for a random node distribution and a random destination for each source?*
- **Transport capacity**
  - Network transports one **bit-meter** when one bit has been transported a distance of one meter
  - Number of bit-meters transported per second
  - *What is the maximum achievable, over all node locations, and all traffic patterns, and all protocols?*

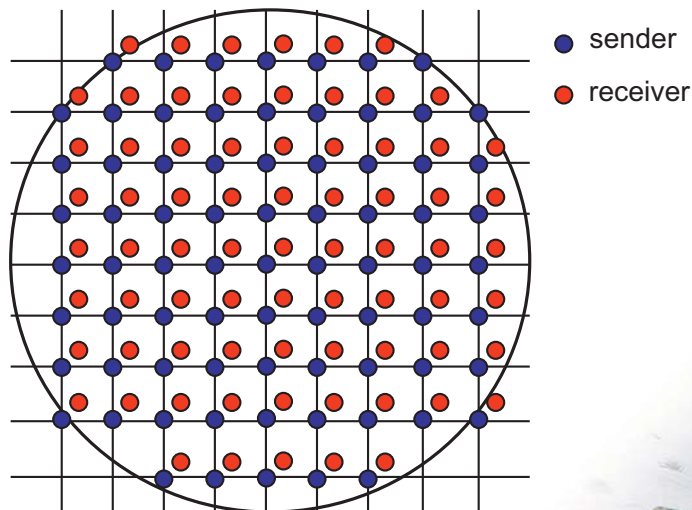


## Transport Capacity

- $n$  nodes are arbitrarily located in a unit disk
- We adopt the **protocol model** with  $R=2$ , that is a transmission is successful if and only if the sender is at least a factor 2 closer than any interfering transmitter. In other words, each node transmits with the same power, and transmissions are in synchronized slots.
- What configuration and traffic pattern will yield the highest transport capacity?
- Idea: Distribute  $n/2$  senders uniformly in the unit disk. Place the  $n/2$  receivers just close enough to senders so as to satisfy the threshold.

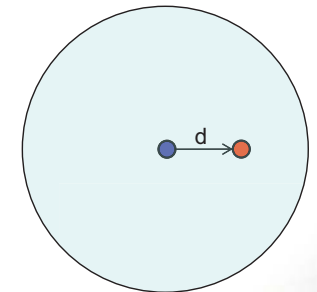


## Transport Capacity: Example



## Transport Capacity: Understanding the example

- Sender-receiver distance is  $\Theta(1/\sqrt{n})$ . Assuming channel bandwidth  $W$  [bits], transport capacity is  $\Theta(W\sqrt{n})$  [bit-meter], or per node:  $\Theta(W/\sqrt{n})$  [bit-meter]
- Can we do better by placing the source-destination pairs more carefully? Not really: Having a sender-receiver pair at distance  $d$  inhibits another receiver within distance up to  $2d$  from the sender. In other words, it kills an area of  $\Theta(d^2)$ .
- We want to maximize  $n$  transmissions with distances  $d_1, d_2, \dots, d_n$  given that the total area is less than a unit disk. This is maximized if all  $d_i = \Theta(1/\sqrt{n})$ . So the example was asymptotically optimal.
  - BTW, a fun geometry problem: Given  $k$  circles with total area 1, can you always fit them in a circle with total area 2?





## More capacities...

- The throughput capacity of an  $n$  node **random network** is  $\Theta\left(\frac{W}{\sqrt{n \log n}}\right)$
- There exist constants  $c$  and  $c'$ , such that  $\lim_{n \rightarrow \infty} \Pr\left[c \frac{W}{\sqrt{n \log n}} \text{ is feasible}\right] = 1$   

$$\lim_{n \rightarrow \infty} \Pr\left[c' \frac{W}{\sqrt{n \log n}} \text{ is feasible}\right] = 0$$
- Transport capacity:
  - Per node transport capacity decreases with  $\frac{1}{\sqrt{n}}$
  - Maximized when nodes transmit to neighbors
- Throughput capacity:
  - For random networks, decreases with  $\frac{1}{\sqrt{n \log n}}$
  - Near-optimal when nodes transmit to neighbors
- In one sentence: **local communication is better...**



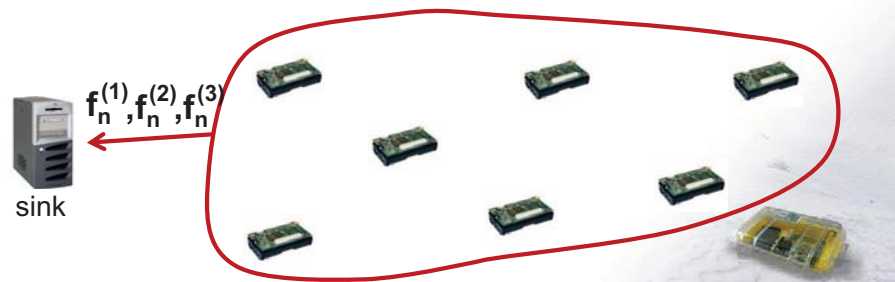
## Even more capacities...

- Similar claims hold in the physical (SINR) model as well...
- Results are unchanged even if the channel can be broken into subchannels
- There are literally thousands of results on capacity, e.g.
  - on random destinations
  - on power-law traffic patterns (probability to communicate to a close-by destination is higher)
  - communication through relays
  - communication in mobile networks
  - etc.
- Problem: The model assumptions are sometimes quite optimistic, if not unrealistic...
- Q: What is the capacity of non-random networks?**



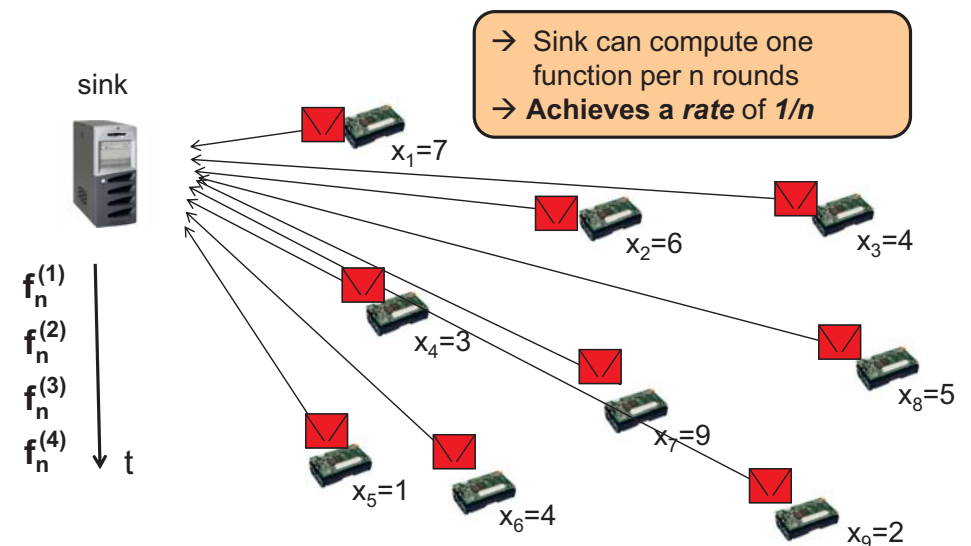
## Data Gathering in Wireless Sensor Networks

- Data Gathering & Aggregation
  - Classic application of sensor networks
  - Sensor nodes periodically sense environment
  - Relevant information needs to be transmitted to **sink**
- Functional Capacity of Sensor Networks
  - Sink periodically wants to compute a **function  $f_n$**  of sensor data
  - At what **rate** can this function be computed?

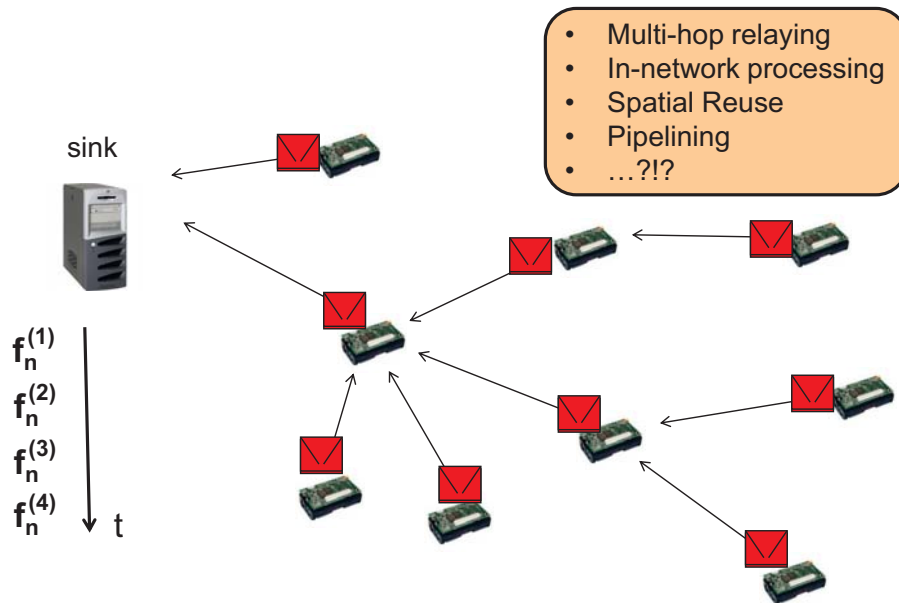


## The Simple Round-Robin Scheme

- Each sensor reports its results directly to the sink (one after another).



## Is there a better scheme?!?



## Capacity in Wireless Sensor Networks

At what **rate** can sensors transmit data to the sink?  
 Scaling-laws  $\rightarrow$  how does rate decrease as  $n$  increases...?

$\Theta(1/n)$      $\Theta(1/\sqrt{n})$      $\Theta(1/\log n)$      $\Theta(1)$

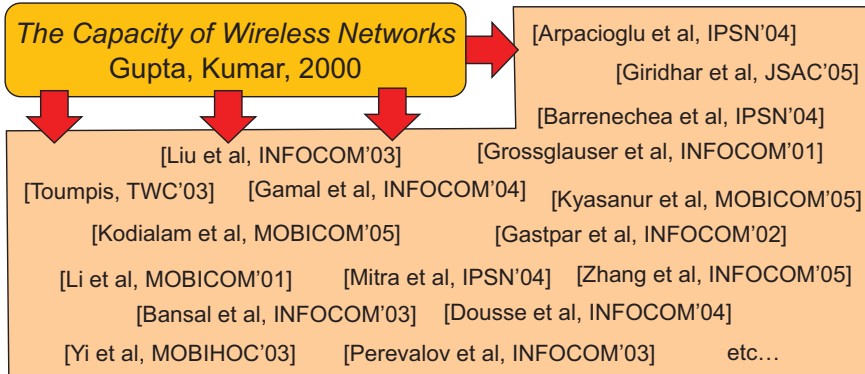
Answer depends on

- Function to be computed  $\rightarrow$  { Only simple functions (max, min, avg,...)
- Coding techniques  $\rightarrow$  { No network coding
- **Network topology**
- **Wireless model**



## Practical relevance?

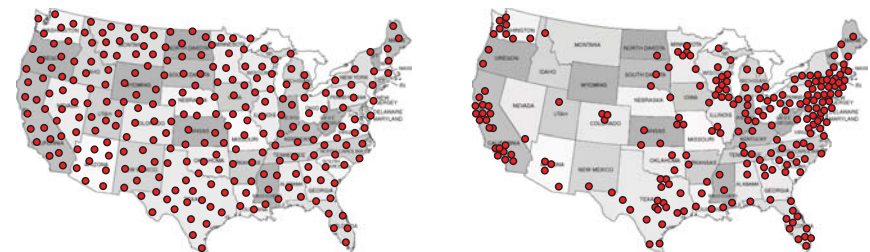
- Efficient data gathering!
- Efficient **MAC** layer!
- This (and related) problem is studied theoretically:



## Network Topology?

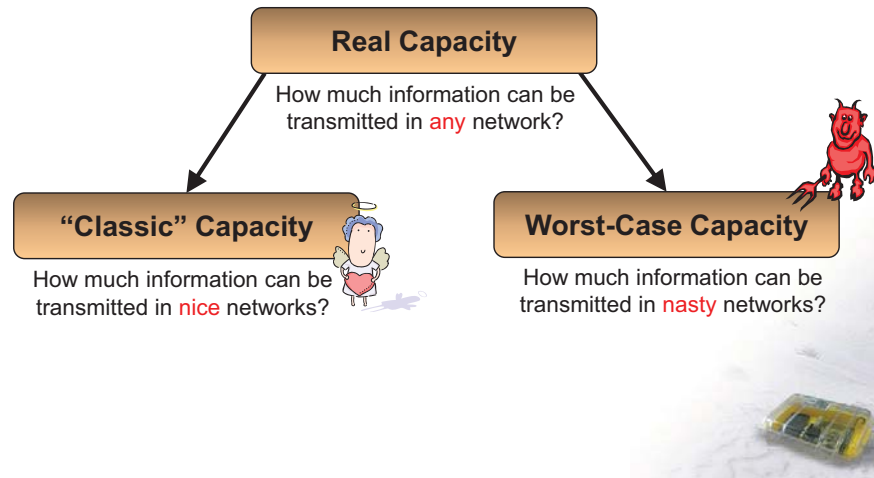
- Almost all capacity studies so far make very **strong assumptions** on node deployment, topologies
  - randomly, uniformly distributed nodes
  - nodes placed on a grid
  - etc.

What if a network looks differently...?



## Capacity for Arbitrary/Worst-Case Network Topologies

- What can one say about worst-case node distributions?
- What can one say about arbitrary node distributions?

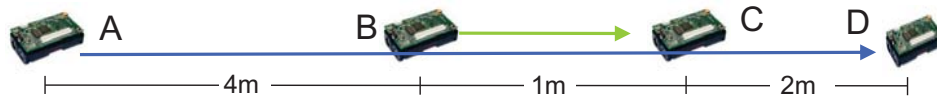


## Wireless Models

- Several models for wireless communication
  - Connectivity-only models (e.g. UDG, QUDG, BIG, UBG, etc.)
  - Interference models
    - Protocol models
      - Two Radii model with constant power (e.g. UDG with interference radius  $R=2$ ).
      - Nodes may use power control (transmission and interference disks of different size)
    - Physical models
      - SINR with constant power (every node transmitting with the same power)
      - SINR with power control (nodes can choose power)
      - Etc.
- Premise: **Fundamental results should not depend on model!**
  - And indeed, classical capacity (assuming e.g. random or regular node distribution) results are similar in all the models above
  - Are there any examples where results depend on model?!

## Simple Example

A sends to D, B sends to C:



Assume a **single frequency** (and no fancy decoding techniques!)



Let  $\alpha=3$ ,  $\beta=3$ , and  $N=10nW$

Transmission powers:  $P_B = -15$  dBm and  $P_A = 1$  dBm

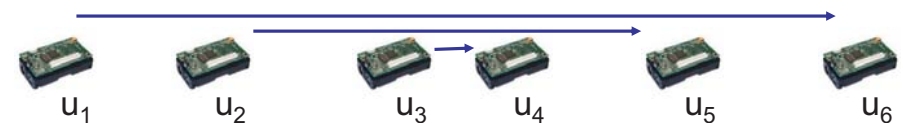
$$\text{SINR of A at D: } \frac{1.26mW/(7m)^3}{0.01\mu W + 31.6\mu W/(3m)^3} \approx 3.11 \geq \beta \quad \text{thumbs up}$$

$$\text{SINR of B at C: } \frac{31.6\mu W/(1m)^3}{0.01\mu W + 1.26mW/(5m)^3} \approx 3.13 \geq \beta \quad \text{thumbs up}$$

## This works in practice!

[Moscibroda et al., Hotnets 2006]

- Measurements using mica2 nodes
- Replaced standard MAC protocol by a (tailor-made) „SINR-MAC“
- Measured for instance the following deployment...



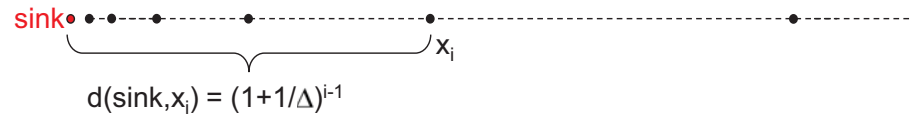
- Time for successfully transmitting 20,000 packets:

	Time required			Messages received	
	standard MAC	„SINR-MAC“		standard MAC	„SINR-MAC“
Node $u_1$	721s	267s	Node $u_4$	19999	19773
Node $u_2$	778s	268s	Node $u_5$	18784	18488
Node $u_3$	780s	270s	Node $u_6$	16519	19498

Speed-up is almost a factor 3

## Worst-Case Rate in Sensor Network: Protocol Model

- Topology (worst-case!): Exponential node chain
- Model: **Protocol model, with power control**
  - Assume for simplicity that the interference radius is twice the transmission radius (however, this can be relaxed easily)



- Whenever a node transmits to another node all nodes to its left are in its interference range. In other words, no two nodes can transmit.
- Network **behaves like a single-hop network!**
- Same result for SINR with constant power or  $P \sim d^\alpha$ .

In the **protocol model**, the achievable rate is  $\Theta(1/n)$ .



## Physical Model with Power Control

In the **physical model**, the achievable rate is  $\Omega(1/\text{polylog } n)$ , independent of the network topology.

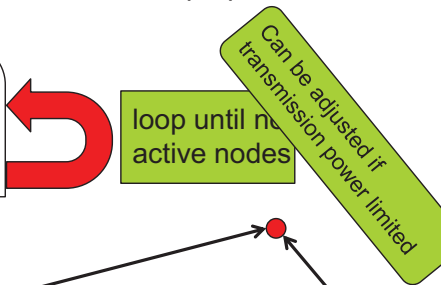
- Original result was  $\Omega(1/\log^3 n)$ . [Moscibroda et al, Infocom 2006]
- Later improved to  $\Omega(1/\log^2 n)$ . [Moscibroda, IPSN 2007]
- Algorithm is centralized, complex  $\rightarrow$  not practical
- But it shows that high rates are possible even in worst-case networks
- Basic idea: Enable **spatial reuse** by **exploiting SINR effects**.



## Scheduling Algorithm – High Level Procedure

- High-level idea is simple
- Construct a hierarchical tree  $T(X)$  that has desirable properties

- 1) Initially, each node is **active**
- 2) Each node connects to **closest active node**
- 3) Break cycles  $\rightarrow$  yields **forest**
- 4) Only root of each tree remains active

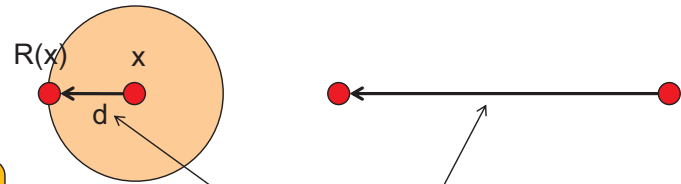


Phase Scheduler:  
How to schedule  $T(X)$ ?

The resulting structure has some **nice properties**  
 $\rightarrow$  If each link of  $T(X)$  can be scheduled at least once in  $L(X)$  time-slots  
 $\rightarrow$  Then, a rate of  $1/L(X)$  can be achieved

## Scheduling Algorithm – Phase Scheduler

- How to schedule  $T(X)$  efficiently
- We need to **schedule links of different magnitude simultaneously!**
- Only possibility:  
senders of small links must **overpower their receiver!**




Subtle balance is needed!

- 1) If we want to schedule both links...  
...  $R(x)$  must be **overpowered**  
 $\rightarrow$  Must transmit at power more than  $\sim d^\alpha$
- 2) If senders of small links overpower their receiver...  
... their "safety radius" increases (spatial reuse smaller)



## Scheduling Algorithm – Phase Scheduler

- 1) Partition links into **sets** of similar length 
- 2) Group sets such that links a and b in two sets in the same group have at least  $d_a \geq (c\beta)^{c(\tau_a-\tau_b)} \cdot d_b$ 
  - Each link gets a  $\tau_{ij}$  value → Small links have large  $\tau_{ij}$  and vice versa
  - Schedule links in these sets in one outer-loop iteration
  - Intuition: Schedule links of similar length or very different length
- 3) Schedule links in a group → Consider in **order of decreasing length** (I will not show details because of time constraints.)

Together with structure of  $T(x) \rightarrow \Omega(1/\log^3 n)$  bound

## Rate in Wireless Sensor Networks: Summary

		Worst-Case Capacity	Traditional Capacity
Power	Networks	Max. rate in arbitrary, worst-case deployment	Max. rate in random, uniform deployment
protocol model or no power control		$\Theta(1/n)$	$\Theta(1/\log n)$
physical model with power control		$\Omega(1/\log^2 n)$	$\Theta(1/\log n)$

[Giridhar, Kumar, 2005]

Exponential gap between protocol and physical model!

The Price of Worst-Case Node Placement

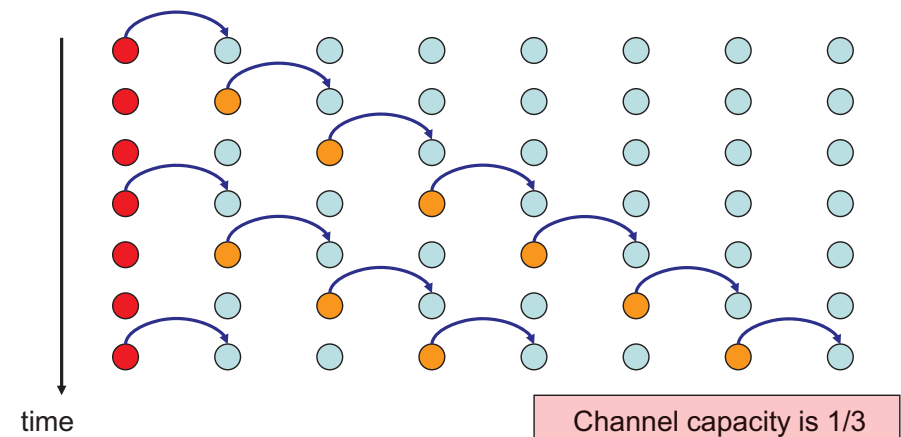
- Exponential in protocol model
- Polylogarithmic in physical model (almost no worst-case penalty!)

## Theoretical Implications

- All MAC layer protocols we are aware of use either uniform or  $d^\alpha$  power assignment.
  - Thus, the theoretical performance of **current MAC layer protocols** is in theory as bad as scheduling every single node individually!
- Faster polylogarithmic scheduling (faster MAC protocols) are theoretically possible in all (even **worst-case**) networks, if nodes choose their transmission power carefully.
  - Theoretically, there is **no fundamental scaling problem** with scheduling.
  - Theoretically efficient MAC protocols **must use non-trivial power levels!**
- Well, the word **theory** appeared in every line... ☺

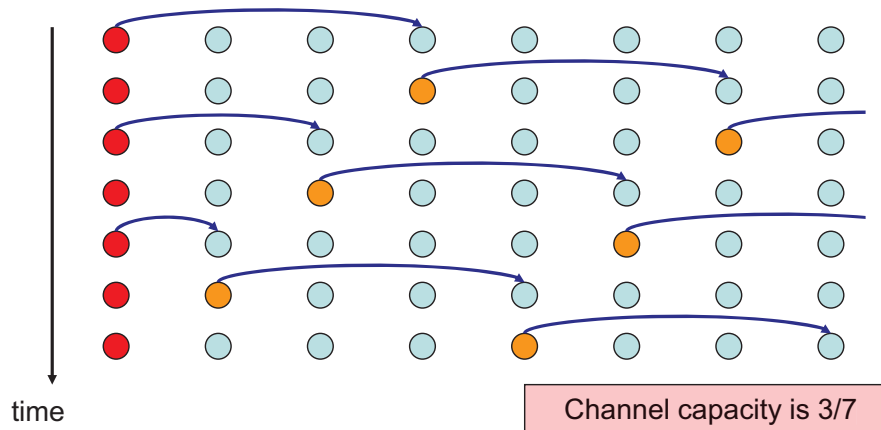
## Possible Applications – Improved “Channel Capacity”

- Consider a channel consisting of wireless sensor nodes
- What is the throughput-capacity of this channel...?



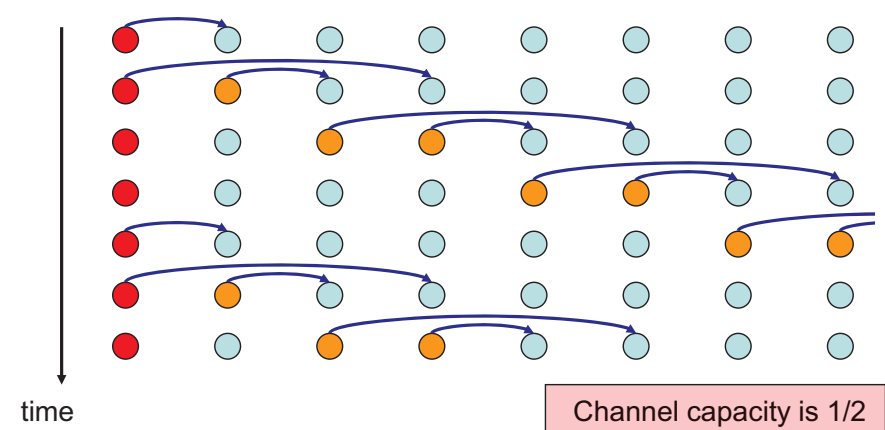
## Possible Applications – Improved “Channel Capacity”

- A better strategy...
- Assume node can reach 3-hop neighbor



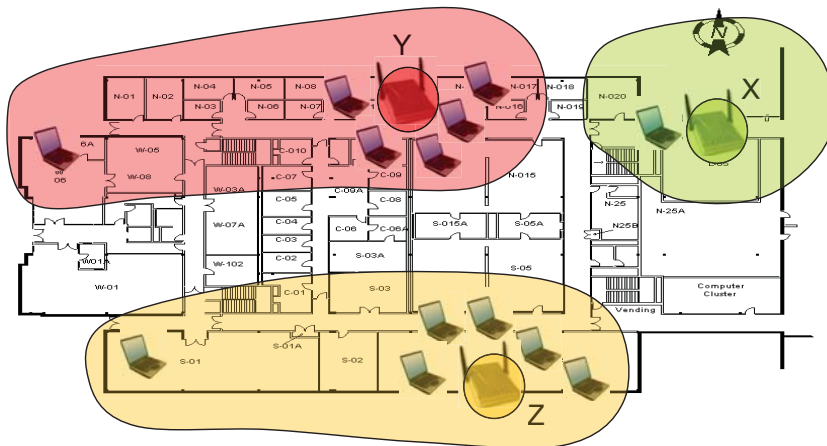
## Possible Applications – Improved “Channel Capacity”

- All such (graph-based) strategies have capacity **strictly less than  $1/2$** !
- For certain  $\alpha$  and  $\beta$ , the following strategy is better!



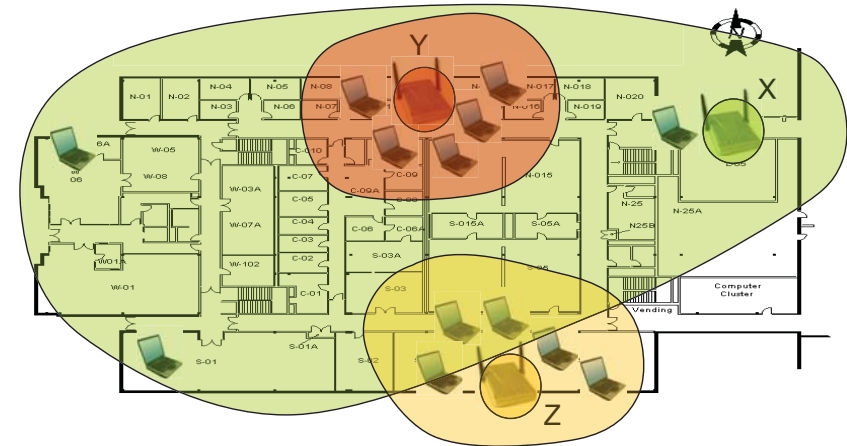
## Possible Application – Hotspots in WLAN

- Traditionally: clients assigned to (more or less) closest access point  
→ far-terminal problem → hotspots have less throughput



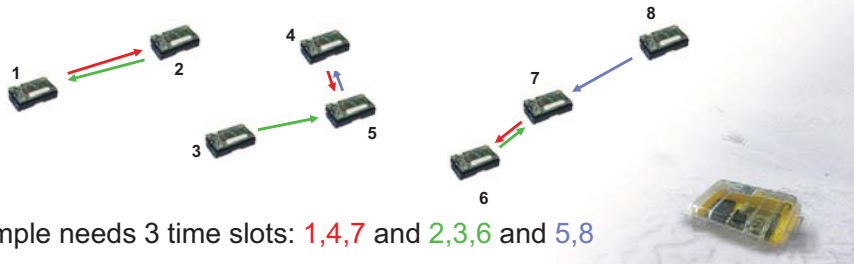
## Possible Application – Hotspots in WLAN

- Potentially better: create hotspots with very high throughput
- Every client outside a hotspot is served by one base station  
→ Better overall throughput – increase in capacity!



## Beyond Worst-/Best-Case: Scheduling Arbitrary Links?

- Given: A set of **arbitrary communication requests** in the plane
  - Each request is defined by position of source and destination
  - Each communication request has the same demand; if some request has a higher demand, just add links between the same sender/receiver
  - Just single-hop, no forwarding at intermediate nodes
  - Model: **SINR with constant power**
- Goal: **Minimize the time to schedule all links!**
  - Those scheduled in the same time slot must obey SINR constraints



Ad Hoc and Sensor Networks – Roger Wattenhofer – 11/41

## Some Results

- Just checking whether some links can be scheduled in the same time slot is trivial. Simply test the SINR at each receiver.
  - In fact, even with power control this is easy. Since distances are fixed the SINR feasibility boils down to a set of linear equations:
 
$$\frac{P_i}{d_i^\alpha} \geq \beta \left( N + \frac{P_1}{d_1^\alpha} + \dots + \frac{P_{i-1}}{d_{i-1}^\alpha} + \frac{P_{i+1}}{d_{i+1}^\alpha} + \dots + \frac{P_k}{d_k^\alpha} \right), P_i \geq 0, \forall i.$$
- On the other hand, scheduling all links in minimum time is difficult (NP-complete), even with constant power. [Goussevskaja et al., 2007]
  - With power control, the complexity of scheduling is still unknown.
- What about **approximation algorithms**? Is it easy to schedule the maximum number of links in one slot? How much time do you need to deliver a given communication demand?
  - Some models are known, others not...

## Definition: Affectance

- The „**affectance**“ of link  $l_v$ , caused by a set of links  $S$ , is the sum of the relative interferences of the links in  $S$  on  $l_v$ . This can even be scaled with noise, using an additional constant  $\eta_v$ . We have:

$$a_{l_v}(S) = \eta_v \cdot \sum_{\ell_w \in S} RI_v(w)$$

with  $\eta_v = \frac{\beta}{1 - \beta \frac{N}{P_{vv}}} = \frac{1}{\frac{1}{\beta} - \frac{N}{P_{vv}}}$  and  $RI_v(w) = \frac{I_{vw}}{P_{vv}}$

Interference of  $w$  on  $v$   
Signal from sender  $v$

- (We simplify by omitting noise. This gives  $\eta_v = \beta$ .)
- A set  $S$  is **SINR-feasible** iff for all  $l_v$  in  $S$  we have  $a_{l_v}(S) \leq 1$ .
- Affectance is additive, i.e.  $a_{l_v}(S_1) + a_{l_v}(S_2) = a_{l_v}(S_1 + S_2)$

Ad Hoc and Sensor Networks – Roger Wattenhofer – 11/43

## One-Slot Scheduling with Fixed Power Levels

- Given a set  $L = \{l_1, \dots, l_n\}$  of arbitrary links, we want to maximize the number of links scheduled in one time-slot
- Constant approximation algorithm:

- Input:  $L$ ; Output:  $S$ ;
- Repeat
  - Add shortest link  $l_v$  in  $L$  to  $S$ ;
  - Delete all  $l_w$  in  $L$ , where  $d_{vw} \leq c \cdot d_{vv}$ ;
  - Delete all  $l_w$  in  $L$ , where  $a_S(l_w) \geq 2/3$ ;
- Until  $L = \emptyset$ ;
- Return  $S$ ;

Schedule “strong” links first

Constant  $c > 2$ ,  $c = f(\alpha, \beta)$

Links with receivers  $s_w$  too close to sender  $r_v$

Links with high affectedness\*

- \*Affectedness has a similar definition as affectance; it also tells how much interference a link can tolerate, i.e.  $a_S(l_v) = 1$  if  $\text{SINR}_S(l_v) = \beta$

Set of links  $S$  is valid iff  $a_S(l_v) \leq 1$  for all  $l_v$  in  $S$

## One-Slot Scheduling: Correctness Proof

- We need to prove affectedness  $a_S(l_v) \leq 1$  for all  $l_v$  in  $S$

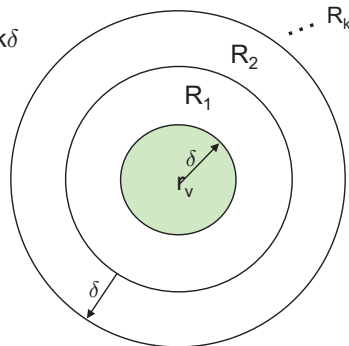
$S_v^-$ : set of shorter ( $\leq$ ) links in  $S$ , i.e., added before  $l_v$

$S_v^+$ : set of longer ( $\geq$ ) links in  $S$ , i.e., added after  $l_v$

$a_{S_v^-}(l_v) \leq 2/3$  (OK! by algo)

$a_{S_v^+}(l_v) \leq 1/3$  (? See below!)

- All senders in set  $S_v^+$  have pair wise distance  $\delta = (c-1)d_{vv}$ .
- We partition the space in infinitely many rings of thickness  $\delta$ .
  - There is no sender in  $S_v^+$  in circle  $R_0$
  - A sender in ring  $R_k$  has at least distance  $k\delta$
  - The number of senders in ring  $R_k$  is  $O(k)$
  - Affectedness from ring  $R_k$  is  $O(\beta k^{1-\alpha} \delta^{-\alpha})$
  - Total affectedness is  $O(\sum_{k \geq 1} \beta k^{1-\alpha} \delta^{-\alpha}) \leq 1/3$ , for  $\alpha > 2$  and large enough constant  $c = f(\alpha, \beta)$



## One-Slot Scheduling: Approximation Proof

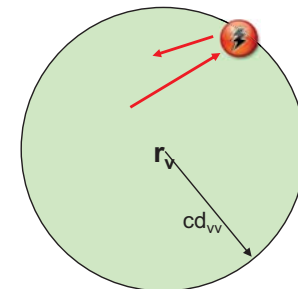
- Count the number of links deleted by ALG that could have been scheduled in the optimum solution **OPT**:  $\text{OPT}' = \text{OPT} \setminus \text{ALG}$

$$\text{OPT}' = \text{OPT}_1 + \text{OPT}_2$$

$\text{OPT}_1$ : links deleted in step 1

$\text{OPT}_2$ : links deleted in step 2

- Claim 1:**  $|\text{OPT}_1| \leq \rho_1 |\text{ALG}|$ , with  $\rho_1 = f(c)$
- Proof:** If the optimal wants to schedule more than  $\rho_1$  links around receiver  $r_v$ , then two of these links have to be very close, and would not satisfy the SINR condition (since their length is at least the length of link  $l_v$ ).

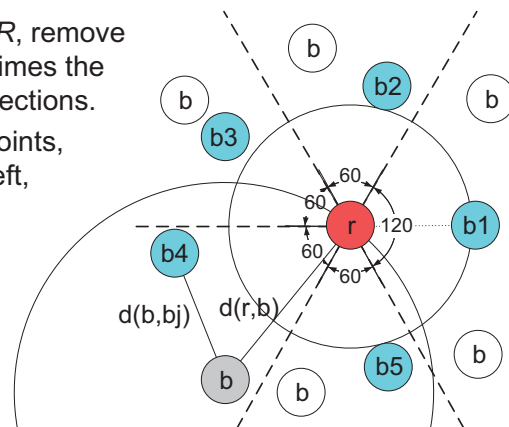


## Helper Lemma: Blue-Dominant Centers Lemma

- Consider two disjoint sets  $B$  and  $R$  of blue and red points in the plane. If  $|B| > 5q|R|$  then at least one point  $b$  in  $B$  is  $q$ -blue-dominant.

**Definition:** a point  $b$  in  $B$  is  $q$ -blue-dominant ( $q$  in  $\mathbb{Z}^+$ ) if, for every ball of radius  $d$  around  $b$ , there are  $q$  times more blue than red points.

- Proof:** for each red point  $r$  in  $R$ , remove  $q$  "guarding sets" of  $r$ , i.e.,  $q$  times the 5 closest blue points in all directions.
- After processing all the red points, at least one blue point  $b^*$  is left, since  $|B| > 5q|R|$ . Point  $b^*$  is blue-dominant, since all red points in  $R$  are "guarded" by at least  $q$  blue points from all directions.



## One-Slot Scheduling: Approximation Proof (Part 2)

- Claim 2:**  $|\text{OPT}_2| \leq \rho_2 |\text{ALG}|$ ,  $\rho_2 = 10$ 
  - Proof:** Let  $B = \text{OPT}_2$  (senders) and  $R = \text{ALG}$  (senders also)
  - Assume for the sake of contradiction that  $|B| > 10|R|$ .
  - By the blue dominant centers lemma, there is a 2-blue-dominant point  $b$  in  $\text{OPT}_2$ . Since  $b$  is 2-blue-dominant, it is twice as much affected by  $\text{OPT}_2$  than by  $\text{ALG}$ . (Really, it is the receiver of link  $l_b$  that is affected but thanks to the clearing of the neighborhood, the receiver is "close" to the sender; some nasty details omitted).
  - In other words, we have  $a_{\text{ALG}}(l_b) < \frac{1}{2} a_{\text{OPT}_2}(l_b)$ . With  $a_{\text{OPT}_2}(l_b) \leq 1$  we have  $a_{\text{ALG}}(l_b) < \frac{1}{2}$ . This **contradicts** that link  $l_b$  has been deleted by step 2 of the algorithm, since step 2 only deletes links with  $a_S(l_b) \geq 2/3$ .
- In summary,  $(\rho_1 + \rho_2) \cdot |\text{ALG}| \geq |\text{OPT}_1| + |\text{OPT}_2| = |\text{OPT}'| = |\text{OPT} \setminus \text{ALG}| \geq |\text{OPT}| - |\text{ALG}|$ . That is,  $|\text{ALG}| \geq |\text{OPT}| / (\rho_1 + \rho_2 + 1) = \Omega(|\text{OPT}|)$ .



## SINR Robustness

- Given a set of scheduling requests  $S$ . We can schedule the set in  $T$  time, iff we can partition the set in  $T$  SINR-feasible subsets  $S_1, \dots, S_T$
- What happens if we change the parameters a bit, e.g. what if we now need to deal with a different  $\beta$ ? Can it be that changing  $\beta$  by some constant factor will lead to a totally different solution?
- In general, what if the world is not perfect?!
  - What if antennas are not perfectly omnidirectional?
  - What if signals are not perfectly summing up?
  - What if signal-to-noise ratio is not perfectly equal to  $\beta$ ?
- Luckily the SINR model is **robust** against minor (constant) changes



## General Robustness Algorithm

- A set  $S$  is **SINR $_{\phi}$ -feasible** iff for all  $l_v$  in  $S$  we have  $a_{l_v}(S) \leq \phi$ . A schedule is **SINR $_{\phi}$ -feasible** if all its slots are SINR $_{\phi}$ -feasible.
  - This generalized feasibility essentially covers all model changes
- From a SINR-feasible schedule to a SINR $_{\phi}$ -feasible schedule:
  1. For each slot  $S$  in a SINR-feasible schedule, process links of  $S$  in decreasing order of their length, i.e.,  $l_1, l_2, \dots, l_k \in S$ , with  $l_v \geq l_{v+1}$ . For each link  $l_v$ , assign  $l_v$  to  $S_j$  with minimum  $j$  with  $a_{l_v}(S_j) \leq \phi/2$ . Then, the affectance on  $l_v$  **by longer links** is at most  $\phi/2$ . By additivity of affectance, the number of sets is at most  $\lceil 2/\phi \rceil$ .
  2. For each set  $S_j$ , do the process in reverse order (short links first), getting sets  $S_{j1}, S_{j2}, \dots, S_{jk}$ . Now, the affectance on  $l_v$  **by shorter links** is at most  $\phi/2$ . Thus, the total affectance is at most  $\phi$  for each link. Also each original set  $S$  is partitioned into at most  $\lceil 2/\phi \rceil^2$  sets.



## SINR Robustness Discussion

- We have constructively shown: If we can schedule a set of links in  $T$  slots in SINR, then we need at most  $O(T/\phi^2)$  slots if we adapt the model by a factor  $\phi < 1$ .
  - This also holds if the scenario is “mixed”, e.g.,
  - if some antennas are a factor  $\phi$  stronger than others, or
  - if the antenna gain in one direction is a factor  $\phi$  stronger, or
  - if walls dampen transmissions by a factor  $\phi$ .
- In other words, if  $\phi$  is constant, the SINR model is **robust** as solutions will continue to be valid up to constant factors.



## Open problem

- This is an area with more open than closed problems. The biggest open problem is **scheduling with power control**. Formally, the problem can be defined as follows:
- A communication request consists of a source and a destination, which are arbitrary points in the Euclidean plane. Given  $n$  communication requests, assign a color (time slot) to each request. For all requests sharing the same color specify power levels such that each request can be handled correctly, i.e., all colors are SINR-feasible. The goal is to minimize the number of colors.
- Pretty much nothing is known about this problem.

