Chapter 7

MAC Theory

7.1 Slide 7/17

Definition 7.1. *An event happens "with high probability" (w.h.p.) if it happens with probability at least $1 - 1/n^c$, for some arbitrary constant c.*

Theorem 7.2. *If nodes wake up in an arbitrary (worst-case) way, any algorithm may take $\Omega(n/\log n)$ time slots until a single node can successfully transmit.*

Proof. Nodes must transmit at some point, or they will surely never successfully transmit. With a uniform protocol, every node executes the same code. We focus on the first slot where nodes may transmit. No matter what the protocol is, this happens with probability p. Since the protocol is uniform, p must be a constant, independent of n.

The adversary wakes up $w := \frac{c}{p} \ln n$ nodes in each time slot, with some constant c. All nodes woken up in the first time slot will transmit with probability p. We study the event E_1 that exactly one of them transmits in that first transmission slot. Using the inequality $(1 + t/n)^n \leq e^t$ we get

$$\begin{aligned} Pr[E_1] &= w \cdot p \cdot (1-p)^{w-1} = c \ln n \cdot (1-p)^{\frac{1}{p}(c \ln n - p)} \\ &\leq c \ln n \cdot e^{-c \ln n + p} = c \ln n \cdot n^{-c} \cdot e^p \\ &= n^{-c} \cdot O(\log n) < \frac{1}{n^{c-1}} = \frac{1}{n^{c'}}. \end{aligned}$$

In other words, w.h.p. that slot will not be successful. Let E_a be the event that all n/w slots will not be successful. Using the inequality $1 - p \leq (1 - p/k)^k$ we get

$$Pr[E_a] = (1 - Pr[E_1])^{n/w} > \left(1 - \frac{1}{n^{c'}}\right)^{\Theta(n/\log n)} > 1 - \frac{1}{n^{c''}}.$$

In other words, w.h.p. it takes more than n/w slots until some node can transmit alone. $\square$