

Discrete Event Systems

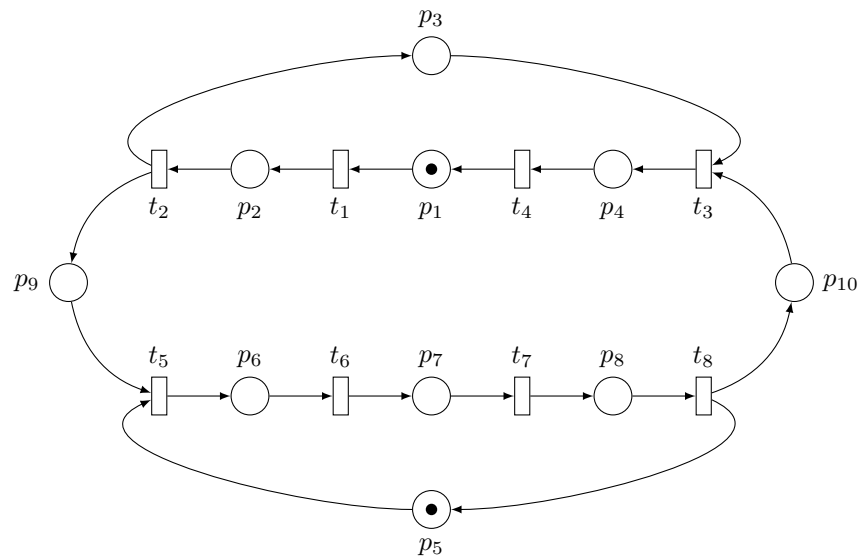
Exercise Sheet 12

1 Labelled Graphs

In the lecture we presented an algorithm that checks the reachability of a state in a directed graph. Given a deterministic *LTS* \mathcal{L} and a word $\omega := w_1w_2 \dots w_n$, modify the algorithm from the lecture such that it checks whether \mathcal{L} accepts the word ω .

2 Structural Properties of Petri Nets and Token Game

Given is the following petri net N_1 :

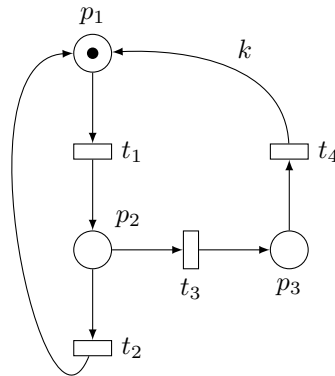


- a) What are the pre and post sets of transitions t_5 and t_8 and of place p_3 ?
- b) Which transitions are enabled after t_1 and t_2 fired?
- c) Determine the number of tokens in N_1 before and after t_2 fired.
- d) Play the token game for N_1 and construct the reachability graph.

Hint: You may denote the states in such a way that the index indicates the places that hold a token in this state, for example $\vec{s}_0 = (1, 0, 0, 0, 1, 0, 0, 0, 0, 0) =: s_{1,5}$.

3 Basic Properties of Petri Nets

Given is the following petri net N_2 :



- a) Explain the terms *boundedness* and *deadlock-freeness* using this example.
- b) For which values of $k \in \mathbb{N}$ is the petri net N_2 bounded/unbounded and not deadlock-free?

4 Reachability Analysis for Petri Nets

In the lecture we presented an algorithm to perform a reachability analysis on petri nets.

- a) Why is it not possible with a reachability algorithm to determine *in general*, whether a given state in a petri net is reachable or not?
- b) Consider the petri net N_2 from exercise 3. Is the state $s = (p_1 = 101, p_2 = 99, p_3 = 4)$ reachable from the initial state $s_0 = (1, 0, 0)$ if $k = 2$? Prove your answer.

Note: Use the condition presented in the lecture that is sufficient for the reachability of a state in a weighted petri net.

5 Mutual Exclusion

Your task is to model a system as a petri net in which two processes want to access a common exclusive resource. This means that the two processes have to exclude each other mutually from the concurrent access to the resource (e.g. a critical program section). More concrete, this means:

1. A process executes its program.
2. In order to enter the critical section, a given mutex variable must be 0.
3. If this is the case, the process sets the mutex to 1 and executes its critical section.
4. When done, it resets the mutex to 0 and enters an uncritical section.
5. Then the procedure starts all over again.