HS 2010 Prof. R. Wattenhofer / J. Smula, T. Langner

Discrete Event Systems

Exercise Sheet 8

1 Colour Blindness/Daltonism

Assume that the average rate of colour blindness is 2/100. (This is a very high rate, but it is supposed to make calculations easier. The actual rate is only about 1/100.000.)

a) Calculate the probability that there are at most three colour blind persons among a random sample of 100 persons.

b) How large does a sample have to be at least such that it contains at least one colour blind person with probability at least 90%?

2 Gloriabar

Every day, 540 students, professors and other personnel of ETH go to the Gloriabar for a *delicious* lunch between 11:45 and 13:15. There is only one queue, and the cashier needs on average 9 s to serve a person. Assume that the arrival and service times are exponentially distributed. Moreover, assume that we do not model the process in which a customer gets its food.

a) Compute the expected waiting time until a student reaches the cashier and the expected time until she has paid for the food.

b) Compute the length of the queue (without the person who is being served).

c) What should be the service time such that the waiting time until a student has paid for her menu is cut in half?

3 Beachvolleyball

The PhD students of the Distributed Computing Group want to participate in a Beachvolleyball tournament. Unfortunately, each DISCO member gets sick sporadically. Assume that the whole DISCO team consists of n players. Further assume that the time period a fit team member remains fit is exponentially distributed with parameter μ (independently of the state of the other players). On the other hand, the time a sick team member remains sick is exponentially distributed with parameter λ.

a) Model the situation as a birth-and-death Markov process where the states denote the number of players that are fit.

b) Derive a formula for the probability that exactly i players are fit, independent of π_0.

Hint:

$$\sum_{i=0}^{n} \binom{n}{i} \cdot x^i = (1+x)^n$$

c) (i) Assume that the DISCO team has $n = 5$ players, and that $\lambda^{-1} = 9$ weeks and $\mu^{-1} = 3$ weeks. Calculate the probability that the DISCO team cannot participate at the tournament. (For participation at least two players are required.)

(ii) How does this probability change if the number of players stays the same, but now $\lambda^{-1} = 4$ weeks and $\mu^{-1} = 2$ weeks?

(iii) For M/M/1 queues, a stationary distribution only exists if ρ is smaller than 1. Why doesn't this hold for this example?

4 Theory of Ice Cream Vending

Apart from their job as PhD students and their nightwatch duties at the Swiss bank, Jasmin and Tobias sell ice cream to further improve their financial situation. In order to serve one customer, each of them needs an amount of time which is exponentially distributed with parameter μ. There is one line of customers in front of their shop, and new customers arrive with a rate λ. Under which conditions is there an equilibrium for this system? And what is the probability that there is no customer in the system (in the steady state)?