Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Distributed Computing

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Discrete Event Systems

Exercise Sheet 11

1 Petri Nets [Exam!]

In this exercise you are supposed to model a function $f_i(x, y)$ on a petri net. That is, the petri net must contain two places P_x and P_y that hold x and y tokens respectively in the beginning. Additionally, the net must contain one place P_z which holds $f_i(x, y)$ tokens when the net is dead. The petri nets are supposed to work for arbitrary numbers of tokens in P_x and P_y.

a) $f_1(x, y) = 5x + y \quad \forall x, y \geq 0$

b) $f_2(x, y) = x - 2y \quad \forall y \geq 0, x \geq 2y$

c) $f_3(x, y) = x \cdot y \quad \forall x, y \geq 0$

Hint: You may have to use inhibitor arcs for part **b)** and **c)**.

2 Competitive Analysis

In this exercise, we analyze algorithms for cellular networks such as GSM. In such networks, the area is segmented into cells, each of which containing a base station. Due to interference, base-stations in neighboring (adjacent) cells cannot use the same carrier frequencies, but frequencies may be reused in non-interfering cells (i.e. cells that are not neighboring). In this exercise, we use the idealized hexagonal grid for modelling these cells (cf. Figure 1).

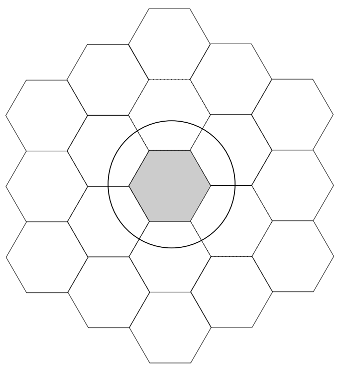


Figure 1: The hexagonal grid modelling the cells.

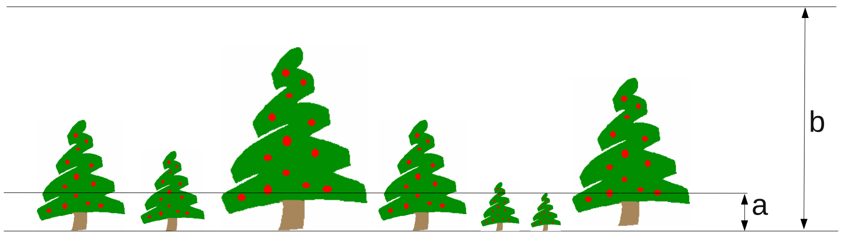
The number of frequencies in real GSM networks is limited. If there are more callers than channels, some calls must be rejected. In this exercise, we make the simplifying assumption that there is only a *single channel* shared by all base stations. That is, every base station can accept exactly one call. Furthermore, if a call is accepted by a base station in a certain cell, no calls can be accepted in neighboring cells due to interference.

Assume that callers arrive in an online fashion, that is, one after another in an input sequence σ. We need to accept or reject callers such that there is at most 1 caller in a cell and its 6 neighboring cells. The *benefit* of the algorithm is the number of calls we accept.

a) Assume that calls have *infinite duration*. Once a call is accepted, it remains active forever. Describe a natural greedy algorithm for the problem. Is your algorithm c-competitive for any fixed constant c? If so, what is the value of c?

b) Assume that every call can have an arbitrary duration, but base stations are not allowed to preempt accepted calls. Propose a competitive online algorithm for this scenario.

3 Competitive Christmas



Roger has been instructed by his children to buy a Christmas tree as large as possible. Nearby his house, a Christmas market sells Christmas trees in a street. Since Roger additionally has to get a lot of presents for Christmas, he does not really have time for searching for a large Christmas tree. But as he passes through this particular street *exactly once* on his optimised shopping route anyway, he intends to get a tree on the way. Being in a hurry, Roger forgot his glasses at home and because he is short-sighted, he can only see the tree right in front of him. Hence, Roger walks through the street and has to decide (online) when to buy a tree (he cannot turn back because then he would not have enough time to get a present for his wife – a disaster). Roger knows from biology classes that the height of fir trees is in the interval $[a, \ldots, b]$ and he knows further that there are n trees in the street.

Help Roger! Give as good a deterministic algorithm as possible that tells Roger when to buy a Christmas tree. How good is your algorithm? What is its competitive ratio?