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Discrete Event Systems

Exercise Sheet 4

1 Regular and Context-Free Languages

a) Consider the context-free grammar G with the production $S \rightarrow SS \mid 1S2 \mid 0$. Describe the language $L(G)$ in words, and prove that $L(G)$ is not regular.

b) The regular languages are a subset of the context-free languages. Give the context-free grammar for an arbitrary language L that is regular.

2 Context-Free Grammars

Give context-free grammars for the following languages over the alphabet $\Sigma = \{0, 1\}$:

a) $L_1 = \{w \mid \text{the length of } w \text{ is odd}\}$

b) $L_2 = \{w \mid \text{contains more 1s than 0s}\}$

3 Pushdown Automata

Consider the following context-free grammar G with non-terminals S and A, start symbol S, and terminals "(", ")", and "0":

$$\begin{array}{lcl} S & \rightarrow & SA \mid \varepsilon \\ A & \rightarrow & AA \mid (S) \mid 0 \end{array}$$

a) What are the eight shortest words produced by G?

b) Context-free grammars can be ambiguous. Prove or disprove that G is unambiguous.

c) Design a push-down automaton M that accepts the language $L(G)$. If possible, make M deterministic.

4 Pumping Lemma Revisited

a) Determine whether the language $L = \{1^{n^2} \mid n \in \mathbb{N}\}$ is regular. Prove your claim!

b) Determine whether the language $L = \{1^{\lfloor \sqrt{n} \rfloor} \mid n \in \mathbb{N}\}$ is regular. Prove your claim!

c) Consider a regular language L and a pumping number p such that every word $u \in L$ can be written as $u = xyz$ with $|xy| \leq p$ and $|y| \geq 1$ such that $xy^iz \in L$ for all $i \geq 0$.

Can you use the pumping number p to give a bound on the minimum number of states needed for the corresponding DFA? What about the minimum number of states of the corresponding NFA?