Eidgenössische Technische Hochschule Zürich
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Distributed Computing

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Discrete Event Systems

Exercise Sheet 6

1 An Unsolvable Problem

It's the first day of your internship at the software firm Bug Inc., and your boss calls you to his office in order to explain your task for the next three months. He says that many clients complain that the programs of Bug Inc. often contain faulty loops that never terminate. In order to prevent such errors in future, you are asked to implement a program that may check whether a given program will halt on all possible inputs or not.

a) Try to find a proof that convinces your boss that this is not possible for general programs.

Hint: The proof works by contradiction. Assume a procedure `halt(P:Program):boolean` that takes a program `P` and decides whether `P` halts on all possible inputs or not. Now construct a program `X` that terminates if `halt(X)` is false and loops endlessly if `halt(X)` is true, which yields the desired contradiction.

b) Your boss still disagrees and proposes the following method: `halt(Y)` simply simulates the execution of program `Y`. If the program terminates it returns true, and if it loops it returns false. Where is the problem of this approach?

c) Your boss is finally convinced but argues that your proof is a very special case that hardly reflects reality. Are there assumptions under which it is always possible to check whether a program halts or not?

2 Dolce Vita in Rome

In order to relax a little bit from the busy life at ETH, Hector and his girlfriend Rachel decide to spend the weekend in Rome. Besides the cultural attractions, Hector and Rachel are also interested in the great choice of ice cream shops (*gelaterie*) that Rome offers.

During their strolls through Rome, the two students encounter n gelaterie. Assume that these ice cream shops can be ranked uniquely according to their attraction, that is, for any two given shops, Hector and Rachel have a clear preference. For instance, the attraction may be a function of the price of the ice cream, quality, atmosphere of the shop, etc.

Since it is too expensive to eat ice cream on every occasion, the two students apply the following strategy: Whenever a shop i is more attractive than the shops 1 to $i-1$ which they have encountered so far, they buy an ice cream.

Assume that the ice cream shops appear in a random order, i.e., any one of the first i shops is equally likely to be the best so far. How many ice creams do Hector and Rachel consume during the weekend in expectation?

3 Soccer Betting

The *FC Basel* soccer club is a particularly moody team. Upon winning a game, they tend to win subsequent games. After losing a game, however, they often end up losing the next game as well. A group of international scientists, consisting of soccer experts, mathematicians, and psychologists, has recently conducted a thorough analysis of this behavior. In particular, they have discovered that upon winning a game, the FCB wins the next game with a probability of 0.6 as well. With probabilities 0.2 each, the next game will be a tie or a loss. After a loss, the FCB will win/tie/lose its next game with probability 0.1/0.2/0.7, respectively. Finally, after a tie, the next game being a win or a loss is equally probable. The probability that the next game also ends up being a tie is 0.4.

a) Model the FCB's moodiness using a Markov chain.

b) In *two games* from now (they will play one game in between), the FCB will play against the FC Zurich. The Swiss TOTO offers you the following odds:

Win: 3.5 Tie: 4.0 Loss: 1.5

Given that the FCB won the game three games ago, but lost the last two games, what would be your bet? Why?

c) More recent studies have shown that the FCB is even moodier than expected. In fact, after losing *two games in a row*, the probability of winning its next game reduces to 0.05, that of getting a tie to 0.1. Change your Markov chain model to incorporate the new findings. How does the change influence your bet?

4 The Winter Coat Problem

While exploring the sea with his boat, Mr. Robinson lost orientation and ended up on a strange island. After living there for several years, he observed that the weather on the island follows a strict probabilistic pattern: The weather of a given day only depends on the weather the day before. When it was sunny (S), the following day is sunny again with 50%, but it gets cloudy (C) with 30% and it starts to rain (R) with 20%. Once its cloudy, it stays cloudy with 10%, gets sunny with 70% and starts to rain with 20%. Finally, after a rainy day, it gets sunny with 10%, cloudy with 40% and remains rainy with 50%.

a) Model this special weather condition using a Markov chain.

b) After spending a sunny day, how many days does Mr. Robinson have to wait until its sunny again (in expectation)?

Due to the global warming of earth, the weather conditions are actually slightly different: After a sunny day, it remains sunny only with 49%, but gets hot (H) with 1%. Once it's hot, it remains hot forever. Similarly, after a rainy day, it remains rainy with 49%, but starts to snow (W) with 1%. Again, once it snows, it continues to snow forever.

c) Adapt your Markov chain to model the new situation.

d) What is the probability that Mr. Robinson ever needs a winter coat, given that he arrived on a sunny day on the island?