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Discrete Event Systems

Exercise Sheet 10

1 Colour Blindness

Assume that the average rate of colour blindness is 2/100. (This is a very high rate, but it is supposed to make calculations easier. The actual rate is only about 1/100.000.)

a) Calculate the probability (exactly, i.e. not using an approximative distribution) that there is at most one colour blind person among a random sample of 100 persons. Calculate it as well using a Poisson-distribution.

b) What is the minimum size of a sample such that it contains at least one colour blind person with probability at least 90%? Now, you should assume that the number of colour blind people is Poisson-distributed with parameter $\lambda = np$ and thereby only obtain an approximative result.

> **The Poisson distribution**
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> The Poisson distribution is a *discrete* probability distribution which is applied often to approximate the binomial distribution for large number n of repetitions and small success probability p of the underlying Bernoulli experiments. According to two frequently used rules of thumb, this approximation is good if $n \geq 20$ and $p \leq 0.05$, or if $n \geq 100$ and $np \leq 10$.
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> The Poisson distribution is often used to estimate the probability of a given number of events occurring in a fixed interval of time and/or space if these events occur with a known average rate and independently of the time since the last event. The parameter $\lambda = np$ of the distribution is the expected number of occurrences in the interval.
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> $$\Pr[X = x] = \frac{\lambda^x}{x!}\, e^{-\lambda}$$

2 Gloriabar

Every day, 540 students, professors and other personnel of ETH go to the Gloriabar for a *delicious* lunch between 11:45 and 13:15. There is only one queue, and the cashier needs on average 9 s to serve a person. Model this situation with an unbounded waiting queue. Assume that the mean interarrival and service times are exponentially distributed and further that we do not model the process in which a customer gets his food.

a) Compute the expected waiting time until a student reaches the cashier and the expected time until she has paid for the food.

b) Compute the expected length of the queue (without the person who is being served).

c) What should be the service time such that the time until a student has paid for her menu is cut in half?

3 Beachvolleyball

The PhD students of the Distributed Computing Group want to participate in a Beachvolleyball tournament. Unfortunately, each Disco member gets sick sporadically. Assume that the whole Disco team consists of n players. Further assume that the time until a fit team member gets sick is exponentially distributed with parameter μ (independently of the state of the other players). On the other hand, the time until a sick team member gets fit is exponentially distributed with parameter λ.

a) Model the situation as a birth-and-death Markov process where the states denote the number of players that are fit.

b) Derive a formula for the probability that exactly i players are fit, independent of π_0.

Hint:

$$\sum_{i=0}^{n} \binom{n}{i} \cdot x^i = (1+x)^n$$

c) (i) Assume that the Disco team has $n = 5$ players, and that $\lambda^{-1} = 9$ weeks and $\mu^{-1} = 3$ weeks. Calculate the probability that the Disco team cannot participate at the tournament. (For participation at least two players are required.)

(ii) How does this probability change if the number of players stays the same, but now $\lambda^{-1} = 4$ weeks and $\mu^{-1} = 2$ weeks?

(iii) For M/M/1 queues, a stationary distribution only exists if ρ is smaller than 1. Why doesn't this hold for this example?

4 Theory of Ice Cream Vending

Apart from their job as PhD students and their nightwatch duties at the Swiss bank, Jochen and Klaus sell ice cream to further improve their financial situation. In order to serve one customer, each of them needs an amount of time which is exponentially distributed with parameter μ. There is one line of customers in front of their shop, and new customers arrive with a rate λ. Under which conditions is there an equilibrium for this system? And what is the probability that there is no customer in the system (in the steady state)?