Eidgenössische Technische Hochschule Zürich
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Distributed Computing

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Discrete Event Systems

Solution to Exercise Sheet 15

1 Network Calculus

Recall that α fulfills the arrival curve property if

$$\forall t\ \forall s : R(t) - R(s) \leq \alpha(t-s) \ ,$$

and β is a service curve if

$$\forall t\ \exists s : R^*(t) - R(s) \geq \beta(t-s) \ .$$

a) If $R \leq R \otimes \alpha$, then by definition for all t:

$$R(t) \leq (R \otimes \alpha)(t) = \inf_u \{R(t-u) + \alpha(u)\}.$$

As this inequality holds for the infimum over all u, it will also hold for *any* u, especially also for $u = t - s$ with *arbitrary* s. With this, we get

$$\begin{aligned} & R(t) && \leq R\left(t-(t-s)\right) + \alpha(t-s) \\ \Longrightarrow\quad & R(t) - R(s) && \leq \alpha(t-s), \end{aligned}$$

which is what we had to show for the first property: The inequality holds for *all* s and t.

b) Similarly, with the definitions from the lecture and the exercise sheet we get for all t

$$R^*(t) \geq (R \otimes \beta)(t) = \inf_u \{R(t-u) + \beta(u)\}.$$

Let u_0 be the u realizing the infimum, and let $s := t - u_0$, i.e. $u_0 = t - s$. Replacing u by u_0 and removing the infimum yields

$$\begin{aligned} & R^*(t) && \geq R\left(t-(t-s)\right) + \beta(t-s) \\ \Longrightarrow\quad & R^*(t) - R(s) && \geq \beta(t-s). \end{aligned}$$

Thus, for all t there exists some $s := t - u_0$ fulfilling the inequality, which is exactly what we had to show.

2 FIFO Calculus

At the beginning of the proof, we assumed that the queue of a node contains s packets. These packets, however, cannot be injected by the adversary at this particular node all at once due to the injection restriction. But the proof tells us how we can get from a node with a queue with k packets to a node with a queue containing k' packets for some $k' > k$ if we choose for an appropriate choide of r and b. Therefore, we can start of with an empty queue, inject a valid number of packets, and repeat the process until we arrive at a queue with s packets for any s. Then, the remainder of the proof holds as shown.