Game Theory in a Nutshell

Notation	Description	Definition
G	finite strategic game	$G = (N, X, U)$
N	set of players	$N = \{1, 2, \ldots, n\}$
X_i	strategy set of player i	
X	set of strategy profiles	$X = X_1 \times X_2 \times \ldots \times X_n$
X_{-i}	set of all other players' strategies of player i	$X_{-i} = X_1 \times \ldots \times X_{i-1} \times X_{i+1} \times \ldots \times X_n$
U_i	payoff function of player i	$U_i : X \to \mathbb{R}$
U	payoff functions	$U = (U_1, U_2, \ldots, U_n)$
$gain(x)$	social gain of outcome $x \in X$	$gain(x) = \sum_{i=1}^{n} U_i(x)$
OPT	social optimum gain	$OPT = \max_{x \in X} gain(x)$
NE	Nash equilibria	$NE = \{x \in X \mid U_i(x) = \max_{x_i \in X_i} U_i(x_i, x_{-i}) \;\; \forall i \in N\}$
PoA	price of anarchy	$PoA = \frac{OPT}{\min_{x \in NE} gain(x)}$
$OPoA$	optimistic price of anarchy	$OPoA = \frac{OPT}{\max_{x \in NE} gain(x)}$
$x_i \succ_d x_i'$	x_i dominates x_i'	$U_i(x_i, x_{-i}) \geq U_i(x_i', x_{-i})$ for every $x_{-i} \in X_{-i}$ and there exists at least one x_{-i} for which a strict inequality holds.
–	x_i^* is dominant strategy	$x_i^* \succ_d x_i$ holds $\forall x_i \in X_i \setminus \{x_i^*\}$
–	$x^* \in X$ is dominant strategy profile	for all players i, x_i^* is the dominant strategy.
$B(x_{-i})$	best responses to x_{-i}	$B(x_{-i}) = \{x_i \in X_i \mid U_i(x_i, x_{-i}) = \max_{x_i' \in X_i} U_i(x_i', x_{-i})\}$
NE	alternative definition of NE	$NE = \{x \in X \mid x_i = B_i(x_{-i}) \;\; \forall i \in N\}$

Dual Definition. For some games it is more natural to describe them with cost functions C_i instead of payoff functions U_i. Consequently, we would define the social cost $cost(x) = \sum_{i=1}^{n} C_i(x)$ of a strategy profile x rather than its social gain, and the definitions of OPT, NE, $(O)PoA$, domination, and $B(x_{-i})$ have to be adapted accordingly. E.g. $OPT = \min_{x \in X} cost(x)$, or $PoA = \max_{x \in NE} cost(x)/OPT$.

Two-Player Games. If $n = 2$ a game can be written as a bi-matrix where the columns correspond to X_1, and the rows correspond to X_2. A field in row a and column b corresponds to a strategy profile where Player 1 plays $a \in X_1$, and Player 2 plays $b \in X_2$. The first number equals $U_1(a, b)$, the second equals $U_2(a, b)$.

Example: Rock, Paper, Scissors. $N = \{1, 2\}$, $X_1 = X_2 = \{rock, paper, scissors\}$.

	rock	paper	scissors
rock	0 , 0	-1 , 1	1 , -1
paper	1 , -1	0 , 0	-1 , 1
scissors	-1 , 1	1 , -1	0 , 0