Eidgenössische Technische Hochschule Zürich
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Distributed Computing

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Discrete Event Systems

Exercise Sheet 5

1 Turing Again [Exam]

a) Construct a TM M that multiplies two positive integers $a, b \geq 1$ (both encoded in unary, i.e., n is encoded with n ones). At the beginning, the tape contains the number a in unary, followed by '$\times$', followed by the number b in unary. The head is positioned over the leftmost '1' of a. After the calculation, the tape should only contain the unary encoded result c (with $c = a \cdot b$), and the head should be positioned over the leftmost '1' of c.

Hint: Empty cells contain the symbol $\square$. You may extend the alphabet if you want.

(i) Describe how your TM M functions in three to four sentences.

(ii) Give a DFA (try to use a small number of states) that operates your TM M. Use the following notation for your transitions ($\gamma = L/R/N$ indicates a direction in which the head should move, i.e. left, right or none):

$\alpha \mid \gamma$	Read α at the current position and then move the head in the direction of γ.
$\alpha \to \beta \mid \gamma$	Read α at the current position, replace it with β, and then move the head in the direction of γ.
$\alpha \to \square \mid \gamma$	Read α at the current position, delete it, and then move the head in the direction of γ.

b) Let M_1 be a TM with the following property: The tape is not infinite in both directions, but just in one direction, i.e., the tape has cells in the range of $[0, \infty)$. Let M_2 be a TM with a normal tape, i.e., infinite in both directions ($(-\infty, \infty)$). Can a TM of type M_1 compute everything that a TM of type M_2 can compute and vice versa? If yes, show how to simulate M_1 on M_2 and the other way around. If not, give a function that can only be computed on one of the two and show why this is the case.

2 An Unsolvable Problem

It's the first day of your internship at the software firm Bug Inc., and your boss calls you to his office in order to explain your task for the next three months. He says that many clients complain that the programs of Bug Inc. often contain faulty loops that never terminate. In order to prevent such errors in future, you are asked to implement a program that may check whether a given program will halt on all possible inputs or not.

a) Try to find a proof that convinces your boss that this is not possible for general programs.

Hint: The proof works by contradiction. Assume a procedure `halt(P:Program):boolean` that takes a program `P` and decides whether `P` halts on all possible inputs or not. Now construct a program `X` that terminates if `halt(X)` is false and loops endlessly if `halt(X)` is true, which yields the desired contradiction.

b) Your boss still disagrees and proposes the following method: `halt(Y)` simply simulates the execution of program `Y`. If the program terminates it returns true, and if it loops it returns false. Where is the problem of this approach?

c) Your boss is finally convinced but argues that your proof is a very special case that hardly reflects reality. Are there assumptions under which it is always possible to check whether a program halts or not?

3 Dolce Vita in Rome

In order to relax a little bit from the busy life at ETH, Hector and his girlfriend Rachel decide to spend the weekend in Rome. Besides the cultural attractions, Hector and Rachel are also interested in the great choice of ice cream shops (*gelaterie*) that Rome offers.

During their strolls through Rome, the two students encounter n gelaterie. Assume that these ice cream shops can be ranked uniquely according to their attraction, that is, for any two given shops, Hector and Rachel have a clear preference. For instance, the attraction may be a function of the price of the ice cream, quality, atmosphere of the shop, etc.

Since it is too expensive to eat ice cream on every occasion, the two students apply the following strategy: Whenever a shop i is more attractive than the shops 1 to $i-1$ which they have encountered so far, they buy an ice cream.

Assume that the ice cream shops appear in a random order, i.e., any one of the first i shops is equally likely to be the best so far. How many ice creams do Hector and Rachel consume during the weekend in expectation?