Eidgenössische Technische Hochschule Zürich
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Networked Systems Group (NSG)

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based on Prof. R. Wattenhoffer's material

# Discrete Event Systems

**Exercise Sheet 4**

## 1 Pumping Lemma Revisited

**a)** Determine whether the language $L = \{1^{n^2} \mid n \in \mathbb{N}\}$ is regular. Prove your claim!

**b)** Consider a regular language $L$ and a pumping number $p$ such that every word $u \in L$ can be written as $u = xyz$ with $|xy| \leq p$ and $|y| \geq 1$ such that $xy^iz \in L$ for all $i \geq 0$.

Can you use the pumping number $p$ to determine the number of states of a minimal DFA accepting $L$? What about the number of states of the corresponding NFA?

## 2 Context Free Grammars

Determine the context free grammar for the following three languages.

**a)** $L_1 = \{w\#x\#y\#z \mid w, x, y, z \in \{a, b\}^* \text{ and } |w| = |z|, |x| = |y|\}$

**b)** $L_2 = \{w \mid \text{the length of } w \text{ is odd}\}$

**c)** $L_3 = \{w \mid \text{contains more 1s than 0s}\}$

*Remark:* Languages $L_2$ and $L_3$ are the same as in Exercice Sheet 3.

## 3 Pushdown Automata: Reloaded

Consider the following context-free grammar $G$ with non-terminals $S$ and $A$, start symbol $S$, and terminals "(", ")", and "0":

$$\begin{aligned} S &\rightarrow SA \mid \varepsilon \\ A &\rightarrow AA \mid (S) \mid 0 \end{aligned}$$

**a)** What are the eight shortest words produced by $G$?

**b)** Context-free grammars can be ambiguous. Prove or disprove that $G$ is unambiguous.

**c)** Design a push-down automaton $M$ that accepts the language $L(G)$. If possible, make $M$ deterministic.

*Remark:* **a)** and **b)** are taken from Exercice Sheet 3.

## 4 Push Down Automata: *The Never Ending Story*

For each of the following context free languages, draw a PDA that accepts $L$.

**a)** $L = \{u \mid u \in \{0, 1\}^* \text{ and } u^{reverse} = u\} = \{u \mid \text{"}u \text{ is a palindrome"}\}$

**b)** $L = \{u \mid u \in \{0, 1\}^* \text{ and } u^{reverse} \neq u\} = \{u \mid \text{"}u \text{ is no palindrome"}\}$