Eidgenössische Technische Hochschule Zürich
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# Distributed Systems Part II 

## Exercise Sheet 8

## Quiz

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## 1 Selling a Franc

Form groups of three to five people. One person is the auctioneer who has to provide one (imaginary) franc. Every other member of the group is a bidder. The franc is allocated to the highest bidder (for his/her last bid). Bids must be a multiple of CHF 0.05. This auction has a crux. Every bidder has to pay the amount of money he/she bid - it does not matter if he/she gets the good. Play the game!
a) Where did it all go wrong?
b) What could the bidders have done differently?

Basic $\qquad$

## 2 Selfish Caching

a) For each of the following caching networks, compute the social optimum, the pure Nash equilibria, the price of anarchy $(P o A)$ as well as the optimistic price of anarchy $(O P o A)$ :
i. $d_{u}=d_{v}=d_{w}=d_{x}=1$

ii. The demand is written next to a node.


## 3 Selfish Caching with variable caching cost

The selfish caching model introduced in the lecture assumed that every peer incurs the same caching cost. However, this is a simplification of the reality. A peer with little storage space could experience a much higher caching cost than a peer who has terabytes of free disc space available. In this exercise, we omit the simplifying assumption and allow variable caching costs $\alpha_{i}$ for node $i$.

What are the Nash Equilibria in the following caching networks given that
i. $\alpha_{u}=1, \alpha_{v}=2, \alpha_{w}=2$,
ii. $\alpha_{u}=3, \alpha_{v}=3 / 2, \alpha_{w}=3$ ?


Does any of the above instances have a dominant strategy profile? What is the PoA of each instance?

## Advanced

## 4 Matching Pennies

Tobias and Stephan like to gamble, and came up with the following game: Each of them secretly turns a penny to heads or tails. Then they reveal their choices simultaneously. If the pennies match Tobias gets both pennies, otherwise Stephan gets them.

Write down this 2-player game as a bi-matrix, and compute its (mixed) Nash equilibria!
Mastery

## 5 PoA Classes

The PoA of a class $\mathcal{C}$ is defined as the maximum $P o A$ over all instances in $\mathcal{C}$. Let

- $\mathcal{A}_{[a, b]}^{n}$ be the class of caching networks with $n$ peers, $a \leq \alpha_{i} \leq b, d_{i}=1$, and each edge has weight 1,
- $\mathcal{W}_{[a, b]}^{n}$ be the class of networks with $n$ peers, $a \leq d_{i} \leq b, \alpha_{i}=1$, and and each edge has weight 1.

Show that $\operatorname{Po} A\left(\mathcal{A}_{[a, b]}^{n}\right) \leq \frac{b}{a} \cdot \operatorname{Po} A\left(\mathcal{W}_{\left[\frac{1}{b}, \frac{1}{a}\right]}^{n}\right)$ for all $n>0$.

