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based on Prof. R. Wattenhofer's material

Discrete Event Systems

Exercise Sheet 4

1 Pumping Lemma Revisited

a) Determine whether the language $L = \{1^{n^2} \mid n \in \mathbb{N}\}$ is regular. Prove your claim!

b) Consider a regular language L and a pumping number p such that every word $u \in L$ can be written as $u = xyz$ with $|xy| \leq p$ and $|y| \geq 1$ such that $xy^iz \in L$ for all $i \geq 0$.

Can you use the pumping number p to determine the number of states of a minimal DFA accepting L? What about the number of states of the corresponding NFA?

2 Context Free or Not?

For the following languages, determine whether they are context free or not. Prove your claims!

a) $L = \{w\#x\#y\#z \mid w, x, y, z \in \{a, b\}^* \text{ and } |w| = |z|, |x| = |y|\}$

b) $L = \{w\#x\#y\#z \mid w, x, y, z \in \{a, b\}^* \text{ and } |w| = |y|, |x| = |z|\}$

3 Push Down Automata

For each of the following context free languages, draw a PDA that accepts L.

a) $L = \{u \mid u \in \{0, 1\}^* \text{ and } u^{reverse} = u\} = \{u \mid \text{“}u \text{ is a palindrome”}\}$

b) $L = \{u \mid u \in \{0, 1\}^* \text{ and } u^{reverse} \neq u\} = \{u \mid \text{“}u \text{ is no palindrome”}\}$

4 Counter Automaton

A push-down automaton is basically a finite automaton augmented by a stack. Consider a finite automaton that (instead of a stack) has an additional *counter* C, i.e., a register that can hold a single integer of arbitrary size. Initially, $C = 0$. We call such an automaton a *Counter Automaton* M. M can only increment or decrement the counter, and test it for 0. Since theoretically, all possible data can be coded into one single integer, a counter automaton has unbounded memory. Further, let $\mathcal{L}_{count}$ be the set of languages recognized by counter automata.

a) Let $\mathcal{L}_{reg}$ be the set of regular languages. Prove that $\mathcal{L}_{reg} \subseteq \mathcal{L}_{count}$.

b) Prove that the opposite is not true, that is, $\mathcal{L}_{count} \nsubseteq \mathcal{L}_{reg}$. Do so by giving a language which is in $\mathcal{L}_{count}$, but not in $\mathcal{L}_{reg}$. Characterize (with words) the kind of languages a counter automaton can recognize, but a finite automaton cannot.

c) Which automaton is stronger? A counter automaton or a push-down automaton? Explain your decision.

5 Designing Turing Machines

Alice is very happy because she was accepted for an internship at Tintel, one of the world's leading processor manufacturers. Unfortunately, she has only attended the famous DES lecture during her studies at ETH and knows nothing about electronic circuits. Therefore, she wants to solve her first assignment using a Turing Machine – please assist her:

Alice is asked to implement a *binary to unary converter*. This converter takes a number a in binary (alphabet $\{0, 1\}$) and converts it to a unary number u (alphabet $\{1\}$). Initially, the TM head points to the MSB of a. At the end, the head should point to the right-most digit of u.

Provide a plain text description of your TM as well as a finite state machine controlling the tape head. Use the following notation for transitions:

'$\alpha \to \beta \mid \gamma$'	read α from the tape at the current position, then write a β and finally move left if $\gamma = L$ or move right if $\gamma = R$.
'$\alpha \mid \gamma$'	abbreviation for transitions of the form $\alpha \to \alpha \mid \gamma$ (these transitions do not modify the content of the tape).

Hint: The number n in unary representation consists of n ones. Also, you might want to extend the alphabet Γ to put temporary symbols on the tape.

6 An Unsolvable Problem

It's the first day of your internship at the software firm Bug Inc., and your boss calls you to his office in order to explain your task for the next three months. He says that many clients complain that the programs of Bug Inc. often contain faulty loops that never terminate. In order to prevent such errors in future, you are asked to implement a program that may check whether a given program will halt on all possible inputs or not.

a) Try to find a proof that convinces your boss that this is not possible for general programs.

Hint: The proof works by contradiction. Assume a procedure `halt(P:Program):boolean` that takes a program `P` and decides whether `P` halts on all possible inputs or not. Now construct a program `X` that terminates if `halt(X)` is false and loops endlessly if `halt(X)` is true, which yields the desired contradiction.

b) Your boss still disagrees and proposes the following method: `halt(Y)` simply simulates the execution of program `Y`. If the program terminates it returns true, and if it loops it returns false. Where is the problem of this approach?

c) Your boss is finally convinced but argues that your proof is a very special case that hardly reflects reality. Are there assumptions under which it is always possible to check whether a program halts or not?