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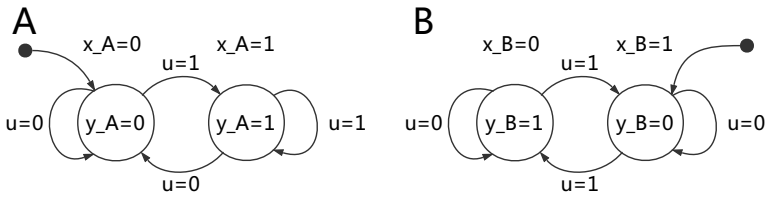
Discrete Event Systems

Exercise Sheet 11

1 Comparison of Finite Automata

Here are two simple finite automata:



For each, we have a one bit encoding for the states (x_A and x_B), one binary output (y_A and y_B), and one common binary input (u). We want to verify whether or not these two automata are equivalent. This can be done through the following steps:

a) Express the characteristic function of the transition relation for both automaton, $\psi_r(x, x', u)$.

b) Express the joint transition function, ψ_f.
Reminder: $\psi_f(x_A, x'_A, x_B, x'_B) = (\exists u : \psi_A(x_A, x'_A, u) \cdot \psi_B(x_B, x'_B, u))$.

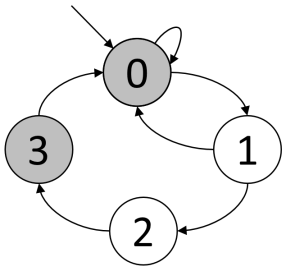
c) Express the characteristic function of the reachable states, $\psi_X(x_A, x_B)$.

d) Express the characteristic function of the reachable output, $\psi_Y(y_A, y_B)$.

e) Are the two automata equivalent? **Hint:** Evaluate, for example, $\psi_Y(0, 1)$.

2 Temporal Logic

a) We consider the following automaton. The property a is true on the colored states (0 and 3).



For each of the following CTL formula, list all the states for which it holds true.

(i) EF a

(ii) EG a

(iii) EX AX a

(iv) EF (a `AND` EX `NOT`(a))

b) Given the transition function $\psi_f(q, q')$ and the characteristic function $\psi_Z(q)$ for a set Z, write a small pseudo-code which returns the characteristic function of $\psi_{\mathrm{AF}\,Z}(q)$. It can be expressed as symbolic boolean functions, like $\overline{x_A}x'_A\overline{x_B}x'_B + \overline{x_A}x'_A x_B x'_B$.
Hint: To do this, simply use the classic boolean operators `AND`, `OR`, `NOT` and ! =. You can also use the operator `PRE`(Q, f), which returns the predecessor of the set Q by the transition function f. That is,

$$\texttt{PRE}(Q, f) = \{q' : \exists q,\, \psi_f(q', q) \cdot \psi_Q(q) = 1\}$$

Hint: It can be useful to reformulate AFZ as another CTL formula.