



Computer Systems

— Solution to Assignment 12 —

1 piChain and PBFT

Quiz

1.1 PBFT: basics

- a) According to Lemma 25.18, it is impossible that two prepared-certificates for the same sequence number are gathered within the same view (not even at different nodes). Therefore, once a node has a prepared-certificate, it can be sure that no correct node will execute a different request for the same sequence number.
- b) The new primary has to send around the new-view-certificate \mathcal{V} ; that certificate has to be valid and the set of **pre-prepared**-messages \mathcal{O} has to be constructed validly from \mathcal{V} in the way specified by the protocol. Since \mathcal{V} already determines the content of \mathcal{O} and the **view-change**-messages in \mathcal{V} are signed, correct replicas can rely on \mathcal{O} if the above conditions hold.
- c) Not necessarily. It is possible that some node u collected a prepared-certificate for a triple (v, s, r) , but as soon as u collected the prepared-certificate, a view change happened. In that case, no correct node can have executed that request yet, but u 's **view-change**-message could still end up in the set \mathcal{V} of the **new-view**-message for the next view.
- d) The proof of Theorem 25.25 shows that if a request was executed by a correct node, then a prepared-certificate will end up in \mathcal{V} . If we take the contrapositive of that statement, we find that if there is no prepared-certificate for a request in \mathcal{V} , then no correct node has executed that request yet. Omitting prepared-certificates for requests that no correct node executed cannot harm correctness of the system.

Basic

1.2 PBFT: we need the phases of the agreement protocol

- a) Backups start their faulty-timer after they receive a request. If backups do not forward requests to the primary, then a faulty client could just send requests to the backups, and the backups' faulty timers would permanently keep expiring, inducing view change after view change.

A byzantine client could make sure to send a request to a backup even without knowing which node is the primary by simply sending distinct requests to all nodes; all but one node

will be backups, and all of their faulty-timers would start running for requests that the primary has never seen and for which the primary can therefore not start the agreement protocol.

- b) Lemma 25.18 implies that two correct nodes cannot agree to execute different requests within a single view, and the proof does not rely on nodes waiting for `commit`-messages, so this Lemma remains intact even with the alteration made in this exercise.

However, the `commit`-messages are important for the view change protocol to maintain safety across views, which we can see in the proof of Theorem 25.25. Consider the following sequence of events:

1. Node u collects a prepared-certificate matching (v, s, r) , and directly executes r . No other node has seen a prepared-certificate yet, and a view change occurs at this moment.
2. The new primary p' of view $v' > v$ collects $2f + 1$ `view-change`-messages, and u 's message is too slow to be included. p' thus does not add a `pre-prepared` (v', s, r, p') $_{p'}$ -message to \mathcal{O} .
3. In the new view v' , correct nodes (with the “help” of byzantine nodes) run the agreement protocol for (v', s, r') for some $r' \neq r$. As soon as correct node $w \neq u$ collects a prepared-certificate matching (v', s, r') , node w will execute r' with sequence number s .

With this, u will execute r with sequence number s , and w will execute $r' \neq r$ with sequence number s .

If $s < s_{max}^v$ (cf. Algorithm 25.23), then r' will be `null`. However, if $s > s_{max}^v$, then r' can be a distinct non-`null` request.

Advanced

1.3 PBFT: multiple prepared-certificates for the same sequence number!? (How does it happen)

In Quiz question c), we saw that a correct node can collect a prepared-certificate that will not be included in a `new-view`-message. Working from this insight, we can imagine the following sequence of events:

1. Only correct node u collects a prepared-certificate for (v, s, r) .
2. A view change to view $v + 1$ happens, and the prepared-certificate that u collected for (v, s, r) is not included in the view change.
3. Correct node w collects a prepared-certificate for $(v + 1, s, r')$ for some $r' \neq r$.
4. A view change to view $v + 2$ happens. Both prepared-certificates are included in the `new-view`-message.

1.4 Authenticated Agreement

- a) We can do roughly the same as we did in Algorithm 25.2, but for multiple values in parallel. Every backup will be collecting messages for every value they hear about. If a correct node gathered agreement for multiple values (or for no values) after $f + 1$ rounds, then it knows that the primary must be faulty. The new algorithm can be seen in Algorithm 1.
- b) The proof is very similar to the one in the script, so we will only give a rough sketch of how to adapt it here:

Algorithm 1 Byzantine Agreement with Authentication

Code for primary p :

- 1: $x \leftarrow$ input value of p
- 2: broadcast $\text{value}(x)_p$
- 3: decide x and terminate

Code for backup b :

- 4: $A \leftarrow \emptyset$
 - 5: **for all** rounds $i \in \{1, \dots, f + 1\}$ **do**
 - 6: **for all** messages $\text{value}(x)_u$ that b received this round **do**
 - 7: $V_x \leftarrow$ {all messages $\text{value}(x)_v$ that b received since round 1}
 - 8: **if** $|V_x| \geq i$ and $\text{value}(x)_p \in V_x$ **then**
 - 9: $A \leftarrow A \cup \{x\}$
 - 10: broadcast $V_x \cup \text{value}(x)_b$
 - 11: **end if**
 - 12: **end for**
 - 13: **end for**
 - 14: **if** $|A| = 1$ **then**
 - 15: decide on the single element in A and terminate
 - 16: **else**
 - 17: decide “sender faulty” and terminate
 - 18: **end if**
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- If the primary is correct, then he only sends one message $\text{value}(x)_p$ in the first round, and all correct backups decide on x after round $f + 1$.
- If the primary is byzantine, then there are these cases:
 1. No correct node ever adds a value to A , then all correct nodes output “sender faulty”.
 2. (The proof of this case is analogous to correct nodes deciding on 1 in the proof in the script. Check the proof in the script if some detail here is unclear.)
At least one correct node adds at least one value x to A . For any value x that gets added to A by some correct node, the first time a correct node adds x to A necessarily happens in a round $i < f + 1$, and all correct nodes will have $x \in A$ in round $i + 1 \leq f + 1$. Since this holds for all x , all correct nodes have the same A after round $f + 1$.
If A contains exactly one value after round $f + 1$, then all correct nodes decide on that value, otherwise all of them decide on “sender faulty”.

Mastery

1.5 PBFT: multiple prepared-certificates for the same sequence number!? (How can we fix it)

Notice that step 2. of the solution to Exercise 1.3 can only occur if the prepared-certificate that u collected was for a request that no correct node executed, see the solution to Quiz question d). This means that we can ignore that prepared-certificate without worrying about it.

On the other hand, for every request that was executed by some correct node, a prepared-certificate for it will end up in every subsequent **new-view-message**, see the proof of Theorem 25.25.

From these considerations, we define the protocol to do the following in Algorithm 25.23: if multiple prepared-certificates for sequence number s end up in \mathcal{V} , then let v^* be the highest view number for which a prepared-certificate (v^*, s, r) exists in \mathcal{V} . The primary p of new view v will include a **pre-prepare** $(v, s, r, p)_p$ -message in \mathcal{O} and ignore all other prepared-certificates for s . The claim we now make is this:

Theorem 1. *Let v^* be the maximum view number of any prepared-certificate for s in \mathcal{V} . If the \mathcal{V} -component of a **new-view**-message contains multiple prepared-certificates for the same sequence number s , then due to Lemma 25.18, there is at most one such prepared-certificate per view v^* . While constructing \mathcal{O} in Algorithm 25.23 during a view change, let the primary of the new view only include a **pre-prepare**-message for the prepared-certificate matching s with the highest view number among the prepared-certificates for s in \mathcal{V} (this is the latest prepared-certificate for s). Then, if some correct node executes a request r with sequence number s in view v , then the latest prepared-certificate for s in \mathcal{V} during every view change after v will match (s, r) .*

Proof. If no **new-view**-message ever contains two prepared-certificates for the same sequence number s but different requests $r' \neq r$, then the Theorem is already proved in the proof of Theorem 25.25 in the script. If no correct node ever executed a request at some sequence number s , then there cannot be a problem with correctness at s either. Thus, assume that a correct node executed request r with sequence number s in view v , and some subsequent **new-view**-message contains multiple prepared-certificates for s . We prove the theorem by induction, showing that once a correct node executed r with sequence number s in view v , then all prepared-certificates for sequence number s that nodes collect in later views will match (s, r) .

Base case: Consider the first view $v' > v$ in which the new primary p' sends a valid **new-view**-message. Since correct backups reject invalid messages, no correct node entered any view v^\dagger with $v < v^\dagger < v'$, so no prepared-certificates were collected in any such view v^\dagger . Thus, v is the highest view number for which any node collected a prepared-certificate for s . As shown in the proof of Theorem 25.25, a prepared-certificate matching (v, s, r) will be in \mathcal{V} in the **new-view**-message for v' .

Induction step: Consider a view change from v' to v'' with $v < v' < v''$ and assume that up to v' , the latest prepared-certificate in the \mathcal{V} -component of **new-view**-messages for s has matched (s, r) . Because backups respond to \mathcal{O} in Algorithm 25.24 before responding to any other **pre-prepare**-messages, nodes can only have collected a prepared-certificate for s with the same r during view v' . Thus, during the view change from v' to v'' , the latest prepared-certificate for s that is in \mathcal{V} will match (s, r) as well. \square

Notice how this clarifies the answer to Exercise 1.3: it can happen that multiple prepared-certificates for the same sequence number s exist in a **new-view**-message. However, if the new primary always picks the latest such prepared-certificate to react to when constructing \mathcal{O} , then that guarantees that once a request r was executed at s by any correct node, then no node will ever be able to gather a prepared-certificate for (s, r') with $r' \neq r$. Thus no correct node will ever execute anything but r at sequence number s .

2 Advanced Blockchain

2.1 Randomness from Previous Block

The validator who has been assigned the hash range starting from 000000... to say, 000000FFFFFFF... has incentive to construct a block such that the hash of that block will land in the their range of hash. In other words, they control the randomness of the process. This way, they will iterate over many combinations of block content till they find the right block that will result in a specific hash that is favorable to them in the next round. QED.