ETH

Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Computer Engineering and
Networks Laboratory

—— TEC, NSG, DISCO ——

HS 2015   Prof. L. Thiele, Prof. L. Vanbever, Prof. R. Wattenhofer

Discrete Event Systems
Exam

Saturday, 6. February 2016, 9:00 – 12:00.

Do not open until told to by the supervisors!

The exam lasts 180 minutes, and there is a total of 180 points. The maximal number of points for each question is indicated in parentheses. Your answers must be in English. Be sure to always justify your answers.

Give your solutions on the page corresponding to the exam question and/or the empty one(s) following it. In case you run out of space, we also added extra pages – in case you use them, please indicate which question you are solving! Should even this not be enough, please contact a supervisor.

Please write down your name and Legi number (student ID) in the boxes below. Once the exam starts, also write your name on every page in the top right corner.

Name	Legi-Nr.

Points

Question	Topic	Achieved Points	Maximal Points
1	Languages, Grammars & Machines		75
2	Delivery Service		18
3	Waste Collectors		26
4	Department of Dawdling		16
5	True or False – Automata & Petri Nets		10
6	BDD		13
7	CTL		10
8	Petri Net		12
Total			**180**

1 Languages, Grammars and Machines (75 points)

1.1 Draw me an automaton [15 points]

Consider the alphabet $\Sigma = \{0, 1\}$ of binary strings representing numbers (with the most significant bit to the left). Draw an DFA which accepts all the strings that are divisible by 5. Strings are read from left to right. For example, the strings 0, 101, 1010 and 01010 would be accepted while 01, 10 would not. Observe that the empty string, ϵ, should not be accepted either.

Hint: Consider relying on the modulo operator (mod) which returns the remainder of the division of one number by another. For instance, $1 \bmod 5 = 1$ and $2 \bmod 5 = 2$.

additional space for **1.1**

1.2 Majority wins [20 points]

Consider 3 different regular languages L_1, L_2, L_3 over the same alphabet Σ recognized by the automatons M_1, M_2, M_3, respectively. Define a "meta-language" L over Σ as follows: $L = \{w \mid w$ is in the majority of $L_1, L_2, L_3\}$. Prove that L is regular by building an automaton $M = (Q, \Sigma, \delta, q_o, F)$ recognizing it based on M_1, M_2, M_3.

a) [10] Explain briefly (in English, with few sentences) how M should be constructed, including a brief description of the set of states, the initial and the final states as well as the transition function;

b) [5] Give a formal description of Q, F, and the initial state q_0 of M;

c) [5] Give a formal description of the transition function δ of M by giving the value of $\delta(q, a)$ for any $q \in Q$ and $a \in \Sigma$.

additional space for **1.2**

1.3 Regular or not? [20 points]

Which of the following L languages are regular? If so, exhibit a finite automaton (deterministic or not) or a regular expression for it. If not, proof it formally using the pumping lemma or the closure properties of regular languages.

a) [5] $L = L_1 \circ L_2$ where $L_1 = \{1^p \mid p \text{ is prime}\}$, $L_2 = \{1^*0\}$ and $\circ$ is the concatenation operator;

b) [5] $L = L_2 \circ L_1$ with L_1 and L_2 defined as above;

c) [5] $L = \{x \# y \mid x, y \in \{a, b\}^* \text{ and } x = y\}$;

d) [5] $L = \{x \# y \mid x, y \in \{a, b\}^* \text{ and } x \neq y\}$. *Hint:* Use the result obtained in **c)** as well as the closure properties.

additional space for **1.3**

1.4 Context-free languages [20 points]

a) [10] Give a context-free grammar (the production rules) for

$$L = \{e^i t^j h^k \mid i, j > 0, k \geq 0, i = 2(j + k)\}$$

b) Let G be the context-free grammar given by the following production rules:

$$S \rightarrow aSb \mid Y$$
$$Y \rightarrow bY \mid Ya \mid \epsilon$$

   (i) [5] What is the language described by the grammar? *Hint:* Your answer should be expressed as a mathematical notation such as the one used in **a)**.

  (ii) [5] Draw a state diagram of a PDA recognizing it.

additional space for **1.4**

additional space for **1**

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2 Delivery Service (18 Points)

Every once in a while, Dr. Disco enjoys having a pizza delivered to his appartment. Recently he noticed that from time to time, he gets a little present with his delivery – extra sauces, sometimes even a garlic bread. Because he's nosy, he asks the delivery man how they decide whether or not a customer gets a present. He is told that the rules to decide this are as follows: Whenever an order comes in, the phone operator of the delivery service rolls a fair 12-sided die ("Würfel"), and she keeps track of the sum of all numbers rolled so far. If the new sum is divisible by 3, the customer gets a small gift; if the sum is divisible by 4, he gets a large gift; if the sum is divisible by 12, she flips a fair coin to decide whether the customer gets a small or a large gift; in all other cases, the customer gets nothing.

a) [6] Model the give-away as a Markov chain.

b) [6] If a small gift is worth 1 CHF and a large gift is worth 5 CHF, what is the expected cost per customer?

c) [3] In expectation, how many customers in a row win large gifts?

d) [3] How would the tie-breaking coin have to be biased to result in equal proportions of customers with small and large gifts?

additional space for **2**

3   Waste Collectors   (26 Points)

Dr. Disco can no longer sustain his luxurious pizza life style and has to resort to collecting waste on the street for money. Luckily, Dr. Disco has a backpack of size B in which he can collect the waste, but as the backpack has a filter against the smell, he cannot take waste out while on a tour.

A tour of waste collecting is performed as follows: Dr. Disco starts at the beginning of a street, inspects each item he passes, decides whether or not to put it into the backpack, and moves on. As his eyesight is bad, he cannot see items ahead or how many items there are in total, and as he is a bit lazy, he won't return to previous items.

- Each item i of waste has a *size* $s_i > 0$ and a *value* $v_i > 0$, which he knows when he looks at the item.

- His goal is to maximize the summed up value of all picked up items in his backpack.

An optimal algorithm works *offline*, i.e., it knows all items beforehand. Throughout this exam question, better competitive ratios or better lower bounds will give more points.

a) [6] Prove/disprove: There is a strictly c-competitive deterministic online algorithm for some $c \in \mathbb{R}$.

From now on (i.e., for **b)**, **c)**, **d)**) , the prize of each item is equal to its size, i.e., $s_i = v_i$.

b) [4] Prove/disprove: There is a strictly c-competitive deterministic online algorithm for some $c \in \mathbb{R}$.

additional space for **3 a) b)**

3 Waste Collectors – continued

Dr. Disco recalls his knowledge from the DES lecture and may now use randomization.

c) [6] Prove a good lower bound for the strict competitiveness of any online algorithm.
Hint: Proving a strictly $(2 - \epsilon)$-competitive lower bound will already give 6 points.

Dr. Disco may now dump all the items in his backpack once at some point, without being allowed to pick them up again.
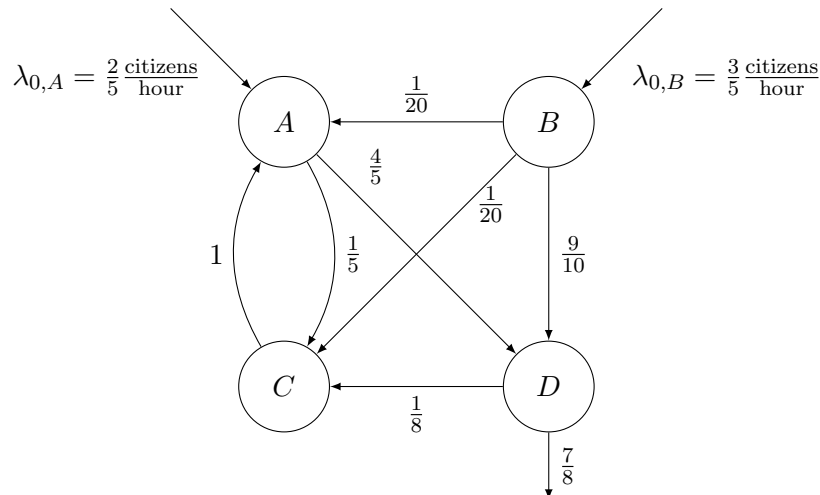
d) [10] Give a strictly c-competitive online algorithm for some $c \in \mathbb{R}$, with or without using randomization.

additional space for **3 c) d)**

4 Department of Dawdling (16 Points)

Some public departments have a reputation of not always being particularly helpful to citizens. Consider the queueing network consisting of M/M/1 queues below; it describes the workings of the Department of Dawdling (DoD), where citizens are sometimes sent around in loops between offices before they finally get to leave. Each of the four offices has a service rate of $2\frac{\text{citizens}}{\text{hour}}$.

$\lambda_{0,A} = \frac{2}{5}\frac{\text{citizens}}{\text{hour}}$ $\quad A \quad$ $\frac{1}{20}$ $\quad B \quad$ $\lambda_{0,B} = \frac{3}{5}\frac{\text{citizens}}{\text{hour}}$

$\frac{4}{5}$ $\quad$ $\frac{1}{20}$

1 $\quad$ $\frac{1}{5}$ $\quad$ $\frac{9}{10}$

C $\quad$ $\frac{1}{8}$ $\quad$ D

$\frac{7}{8}$

a) [7] Three employees of the DoD have a disagreement: one of them thinks that on average, less than 1 citizen is inside the DoD, the second thinks it's exactly 1, and the third that it's more than 1. Who is right?
Hint: This can be answered without solving systems of linear equations.

b) [9] If a citizen arrives at office A while no one else is visiting the DoD, and no one arrives while she is in the DoD, how long does it take her in expectation to leave the DoD again?

additional space for **4**

additional space for **2,3,4**

# 5   True or False – Automata & Petri Nets   (10 points)

In this question, every correct answer will give 1 point, and every false answer will give 0 points.
Please write *True* or *False* to the left of each question, no justification is needed.

- [1] Assuming A, B, C and D are sets and ψ_X is the boolean representation of set X,
  $(A \cup B) \cap (C \backslash D) \quad \Leftrightarrow \quad \psi_A \cdot \psi_B + \psi_C \cdot \overline{\psi_D}$

- [1] Given an ordering of variables, the OBDD of a boolean function is unique.

- [1] The number of nodes of an OBDD is independent of the ordering of variables.

- [1] $A(EF\phi)$ is a valid CTL formula.

- [1] $EG\left(\overline{\phi} A U \psi\right)$ is a valid CTL formula.

- [1] If the transition of a Petri net is L_1-live, then it is L_4-live.

- [1] If a Petri net marking is reachable from a given initial state, it is always possible to reach it from another state.

- [1] The following Petri net has two different states.
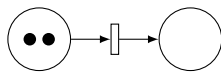
- [1] There exists a Petri net with an infinite number of states.

- [1] The following timed Petri net has a delay of 10 time units on transition T.
  According to the definition from the lecture, T can be fired anytime after 10 time units.
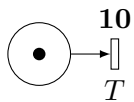
6   BDD   (13 points)

You are given the following boolean functions:

$$f_1 : y = \overline{(x_1 + \overline{x_3}) \cdot (x_2 + x_4)} + (x_2 + x_3)$$

$$f_2 : y = x_3 + x_1 \cdot x_2 + x_1 \cdot \overline{x_2} \cdot x_4 + \overline{x_1} \cdot \overline{x_2} \cdot \overline{x_4}$$

Considering the ordering of variables $\Pi : x_1 < x_2 < x_3 < x_4$ (i.e., x_1 is the first variable):

a) [6] Construct the BDD of f_1. Merge all equivalent nodes.

b) [6] Construct the BDD of f_2. Merge all equivalent nodes.

c) [1] Are f_1 and f_2 equivalent? Why?

additional space for **6**

7 CTL   (10 points)

a) Given the automaton in Fig.1, for each case give the set of states for which the property holds. We assume the atomic property p is true in states 0 and 2 only.
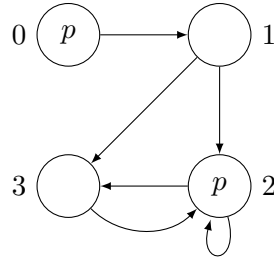

Abbildung 1: A simple automaton

(i) [1] EGp

(ii) [1] $EG\overline{p}$

(iii) [1] AGp

(iv) [1] EFp

(v) [1] AFp

(vi) [1] $AF\overline{p}$

(vii) [1] EXp

(viii) [1] $AX\overline{p}$

(ix) [1] $EX\,(AXp)$

b) [1] Reformulate the property EGp using the A, F, and $\overline{}$.

additional space for **7**

8 Petri Net   (12 points)

8.1 About coverability [5 points]

a) [3] Construct the coverability tree of the Petri net from Fig.2.

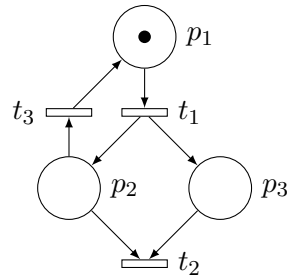**b)** [2] Deduce its coverability graph.

Abbildung 2: Petri net for 8.1

additional space for **8.1**

8.2 About reachability [7 points]

a) [3] What is the incidence matrix of the Petri net from Fig.3?

b) [2] From the initial marking M_0, what is the marking M_1 obtained by firing the sequence $S = t_1 t_2 t_3$?

c) [2] Does there exist a marking M_i from which you can reach the marking M_0 after firing the same sequence S?
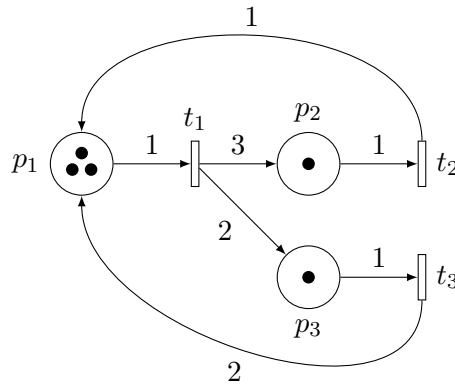

Abbildung 3: Petri net for 8.2

additional space for **8.2**

additional space for **6,7,8**