Eidgenössische Technische Hochschule Zürich
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Networked Systems Group (NSG)

HS 2020

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based on Prof. R. Wattenhofer's material

Discrete Event Systems

Exercise Sheet 5

1 Minimum Pumping length

Consider the regular language $L = 1^*0^+1^+0^* \cup 111^+0^+$. Give the minimum pumping length and briefly explain the intuition behind your answer.

2 The art of being regular

Assume that the alphabet Σ is $\{0, 1\}$ and consider the language $L = \{x\#y \mid x + y = 3y\}$ in which x and y are binary numbers. For instance, the string 1000#100 belongs to L. Is L regular? If so, exhibit a finite automaton (deterministic or not) or a regular expression recognizing it. If not, prove it formally using the pumping lemma or the closure properties of regular languages.

3 Counter Automaton

A push-down automaton is basically a finite automaton augmented by a stack. Consider a finite automaton that (instead of a stack) has an additional *counter* C, i.e., a register that can hold a single integer of arbitrary size. Initially, $C = 0$. We call such an automaton a *Counter Automaton* M. M can only increment or decrement the counter, and test it for 0. Since theoretically, all possible data can be coded into one single integer, a counter automaton has unbounded memory. Further, let $\mathcal{L}_{count}$ be the set of languages recognized by counter automata.

a) Let $\mathcal{L}_{reg}$ be the set of regular languages. Prove that $\mathcal{L}_{reg} \subseteq \mathcal{L}_{count}$.

b) Prove that the opposite is not true, that is, $\mathcal{L}_{count} \nsubseteq \mathcal{L}_{reg}$. Do so by giving a language which is in $\mathcal{L}_{count}$, but not in $\mathcal{L}_{reg}$. Characterize (with words) the kind of languages a counter automaton can recognize, but a finite automaton cannot.

c) Which automaton is stronger? A counter automaton or a push-down automaton? Explain your decision.