Discrete Event Systems

Exercise session #5

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1. Minimum Pumping length

Give the minimum pumping length of the regular language L.
Briefly explain the intuition behind your answer.

$$L = \underbrace{1^*0^+1^+0^*}_{L1} \cup \underbrace{111^+0^+}_{L2}$$

Assume p1 is the minimum pumping length of L1
p2 is the minimum pumping length of L2
p is the minimum pumping length of L

1. Minimum Pumping length

If there is no single word that belongs to both L1 and L2

$$p <= max\{p_1, p_2\}$$

1. Minimum Pumping length

Assume p1 = 3

- s = 101, then it can be divided into xyz where y =1
- s = 010, then it can be divided into xyz where y =0

The minimum pumping length for L1 is thus 3.

1. Minimum Pumping length

Assume p2 = 4

- s = 1110 cannot be pumped, thus the minimum pumping length of L2 cannot be 4.

Assume p2 = 5

- s = 11110, then it can be divided into xyz where x=111, y=1 and z=0 and thus can be pumped.
- s = 11100, then it can be divided into xyz where x = 111, y = 0 and z = 0 and thus can be pumped.

Thus p2= 5.

1. Minimum Pumping length

$$p <= max\{p_1, p_2\}$$

2. The art of being regular

Assume the alphabet $\Sigma = \{0, 1\}$ and the language

$$L = \{x\#y \mid x + y = 3y\}$$

in which x and y are binary numbers.

For instance, the string 1000#100 belongs to L.

If so, exhibit a finite automaton (deterministic or not) or a regular expression recognizing it. If not, prove it formally using the pumping lemma or the closure properties of regular languages.

2. The art of being regular

L is not regular.

Consider w=100^p #10^p.

Then w $\in$ L since x = 100^p = 2y , where y I= 10^p for p>=0.

2. The art of being regular

L is not regular.
Consider w=100^p #10^p.
Then w ∈ L since x = 100^p = 2y , where y I= 10^p for p>=0.

We must consider three cases for where y can fall:

a) y=1. If i=0 arithmetic is wrong: the left side is 0 but right side isn’t.
b) y=10* Arithmetic is wrong.
c) y = 0^p. If i =0 arithmetic is wrong: decreased left side but not right.