Discrete Event Systems HS2020

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1 First Bonus Task

Prove that $L = \{w0\#w \mid w \in 1(0 \cup 1)^*\}$ *is not context-free.*

We solve this task using the tandem-pumping lemma. First we assume for contradiction that L is context-free and hence there is a number p such that any string in L of length $\geq p$ is tandem-pumpable within a substring of length p. We choose $w = 1^p0^p$ and hence the word we consider is $\alpha = w0\#w = 1^p0^p0\#1^p0^p$ with $|\alpha| \geq p$.

We now want to split $\alpha = uvxyz$ with $|vy| \geq 1$, $|vxy| \leq p$ and $uv^ixy^iz \in L$ for all $i \geq 0$. Because we have $|vxy| \leq p$, there are the following options:

- $\# \notin vxy$ ($vxy = 1^m$ or $vxy = 0^m$ with $1 \leq m \leq p$ or $vxy = 1^n0^s$ with $n + s \leq p$). Any one of these sequences can either be before or after the # but independent of this choice, if we pump v and y and choose for example $i = 0$, we will have $\alpha' = w'0\#w''$ with $w' \neq w$ and hence $\alpha' \notin L$.
- $\# \in vxy$. In this case, we can choose $x = \#$ because we know that there is only one # and therefore this cannot be the pumpable part. This leaves us with $v = 0^n$ and $y = 1^s$ with $1 \leq n + s \leq p - 1$ and if we for example set $i = 0$ this leaves us with $\alpha' = 1^p0^{p+1-n}\#1^{p-s}0^p$ which is $\notin L$.

Because we have now considered all possible splits of this word into $\alpha = uvxyz$, we can safely say that language L is not context-free.

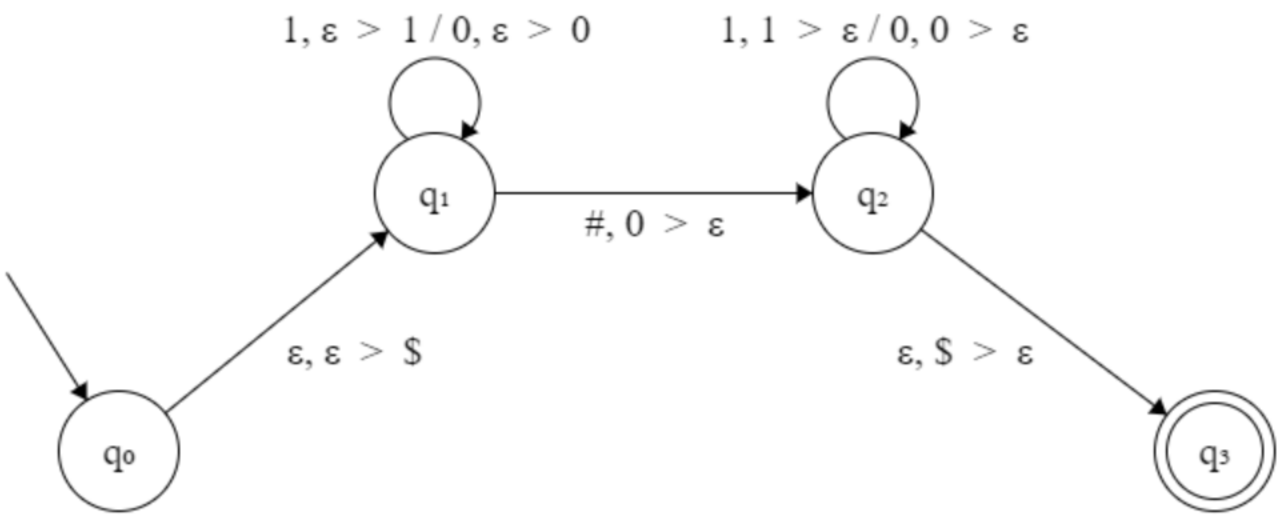
2 Second Bonus Task

Prove that $L = \{x\#y \mid x + reverse(y) = 3 \cdot reverse(y)\}$ *is context-free.*

Applying the same transformations as we did in the exercise class with $w' = reverse(w)$, we arrive at $L = \{x\#y \mid x = 2 \cdot reverse(y)\} = \{w0\#w' \mid w \in 1(0 \cup 1)^*\}$. We can show that this language is context-free by drawing a push-down automaton that accepts this language. This automaton is pictured on the next page with > representing stack operations $\rightarrow$.

We could alternatively also show that the language is context-free by providing a context free grammar (V, Σ, R, S) such as the following:

- $V = \{S\}$
- $\Sigma = \{0, 1, \#\}$
- $R: \ S \rightarrow 1S1 \mid 0S0 \mid 0\#$
- $S = S$


1, ε > 1 / 0, ε > 0
1, 1 > ε / 0, 0 > ε
q1
q2
#, 0 > ε
ε, ε > $
ε, $ > ε
q0
q3