Eidgenössische Technische Hochschule Zürich
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Distributed Computing

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Computational Thinking
Exercise 13

1 Undecidable problems

Prove that the following problems are undecidable.

a) Given a TM T' (in some encoding), an input x to T', and a symbol $\sigma \in \Sigma$, does T' ever write the symbol σ on the tape while running on x?

b) Given two TMs T_1 and T_2, is there an input x that T_1 halts on x, but T_2 does not?

2 Turing Machines

Describe a Turing Machine that solves the following simple computational problems. You can assume that the input alphabet contains $\Sigma = 0, 1$, the empty symbol $\perp$, plus arbitrary special symbols of your choice.

a) Given an input string, decide if the string is a palindrome (i.e. if it gives the same string if you read it in the other direction, from back to front).

b) Given two binary numbers (separated by a special character '\$'), compute the sum of the two numbers. You can assume for convenience that the bitstrings representing the two numbers have the same length: if one of them consists of fewer bits, then some padding 0's are added to the front to ensure that they consist of the same number of bits.

Note that the online notebook gives you a framework where you can formally code the TMs for these exercises, and test the correctness of your solution.

3 One-dimensional tiling

Consider the one-dimensional version of the tiling problem: each tile only has a color on its left and right side, and our job is to create an infinitely long horizontal line with a correct tiling (i.e. where touching sides always have the same color).

a) Show that if there exists a solution to the one-dimensional tiling problem with a set of tiles S, then there also exists a periodic solution with this set of tiles S.

b) Prove that the one-dimensional tiling problem is decidable (i.e. solvable in finite time for any tile set S).