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Principles of Distributed Computing
Exercise 1

1 Vertex Coloring

a) In the lecture, a simple distributed algorithm which colors an arbitrary graph with $\Delta + 1$ colors in n synchronous rounds was presented (Δ denotes the greatest degree, n the number of nodes of the graph). To run in n rounds, the nodes of the graph had to be numbered from 1 to n. Devise a synchronous distributed algorithm for the case the IDs are unique but unbounded numbers (i.e. the nodes have arbitrary IDs instead of being numbered from 1 to n). Your algorithm should also use at most $\Delta + 1$ colors and terminate in a linear number of synchronous rounds.

b) What is the total number of messages your algorithm sends?

c) Does your algorithm also work in an asynchronous environment? If yes, formulate the asynchronous equivalent to your algorithm, if no, describe why.

2 Counting the Nodes of a Tree

In this exercise, we assume that the communication graph T is a tree. We consider different aspects of the problem of counting the number of nodes of T.

a) Suppose that a node $v \in T$ wants to know the total number of nodes. Develop a distributed algorithm $\mathcal{A}$ for this task. $\mathcal{A}$ can be started by every node v of T, it should determine the number of nodes of T and report it to v. How long does your algorithm need until v knows the result?

b) Suppose now that all nodes would like to know the number of nodes in T. Devise an algorithm with which all nodes of a tree T concurrently calculate the number of nodes.

c) For the last question of this exercise, we assume that the tree T has an odd number of nodes. In such a tree, there is a unique node v which allows to divide T into two parts whose sizes are as equal as possible. Can you use your results of Question 2b) to develop a distributed algorithm to find v? What can you say about the sizes of both parts of the achieved partition of T?