



# **Topology Control in Heterogeneous Wireless Ad-hoc Networks**

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# Overview of this Presentation

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- Part 1:
  - Introduction
  
- Part 2:
  - Paper1: General Graphs
    - DRNG & DLMST
  - Paper2: Mutual Inclusion Graphs
    - $EYG_k(MG)$
  
- Part 3:
  - Proof for connectivity in DLMST
  - Conclusions



# About the Papers

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- Topology Control in Heterogeneous Wireless Networks: Problems and Solutions
  - Ning Li and Jennifer C. Hou
  - INFOCOM 2004
- Localized Topology Control for Heterogeneous Wireless Ad-hoc Networks
  - Xiang-Yang Li, Wen-Zhan Song, Yu Wang
  - MASS 2004



# Part 1

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## Introduction



# Why Topology Control?

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- maintain network connectivity
  - every node can reach all others
  - reduce energy consumption
  - sending over near neighbours is more efficient than sending directly to a far target
  - do not send with maximal transmission power if not necessary
- improve network capacity



# Related Work

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- based on centralised algorithms
  - applicable for static networks
  - need global information
  - can achieve optimality
- based on unit disk graphs
  - homogeneous wireless nodes with uniform transmission ranges
  - every node sends with same transmission power
- based on fixed nodes
  - once a node has been initialised, it does not change its position



# Why Heterogeneous Networks? (1)

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- can easily add new devices without attention to the type of the device (mobility, dynamic)
  - we can use devices with non-uniform transmission ranges
- in practice there are many influences which affect the range of a device
  - obstacles like plants, walls, ... or other radio frequencies



## Why Heterogeneous Networks? (2)

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- there exist heterogeneous networks in which devices have dramatically different capabilities
  - Military: devices on soldiers vs. devices on vehicles
- even devices of the same type may have slightly different maximal transmission power





# What we want

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- each wireless node should locally
  - adjust its transmission power
  - select with which neighbours to communicate
- model should deal with dynamic changes in topology
  - addition of new nodes
  - removal or drop out of links (or nodes)



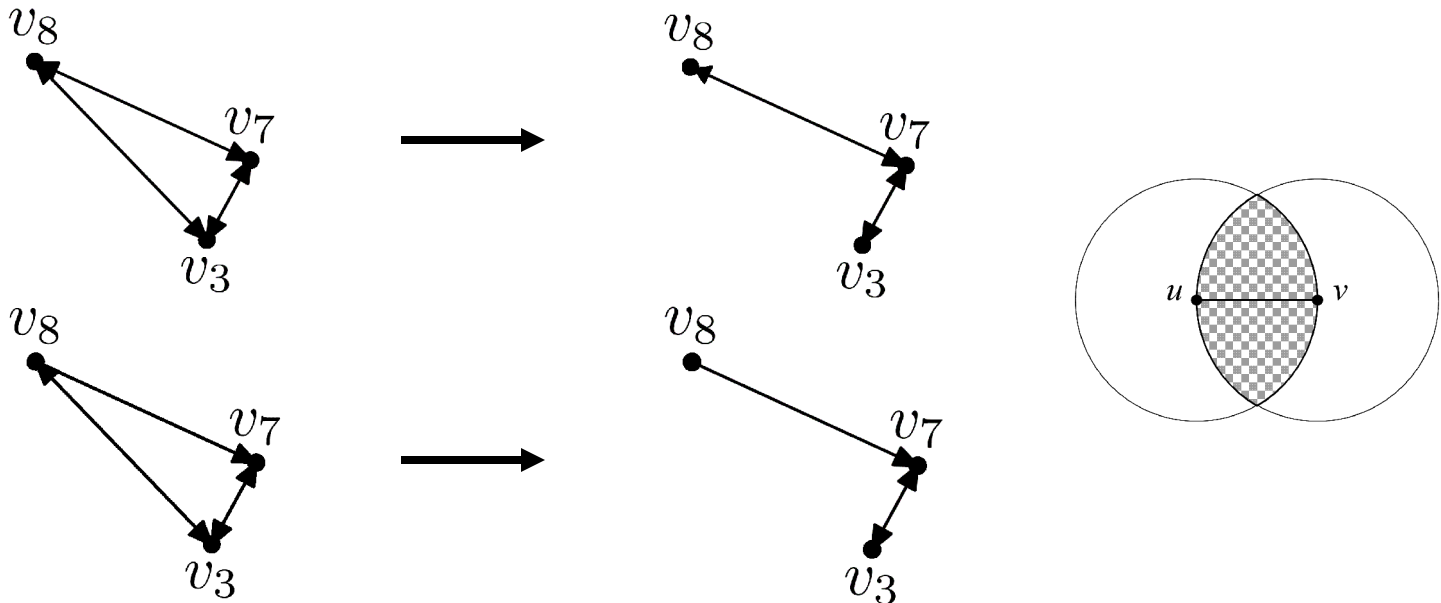
# Simple adaptation doesn't work (1)

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- can't guarantee network connectivity in heterogeneous case
  - no global information
  - assumptions about transmission power of counterparts don't hold anymore
- message overhead
  - energy
- unbounded out-degree
  - increase signal interference & overhead at a node

# Simple adaptation doesn't work (2)

- a RNG structure in a homogeneous graph is connected since all links would be bi-directional
- edge  $(v_3, v_8)$  is discarded since  $v_7$  lies in the shaded area between  $v_3$  and  $v_8$





## Part 2

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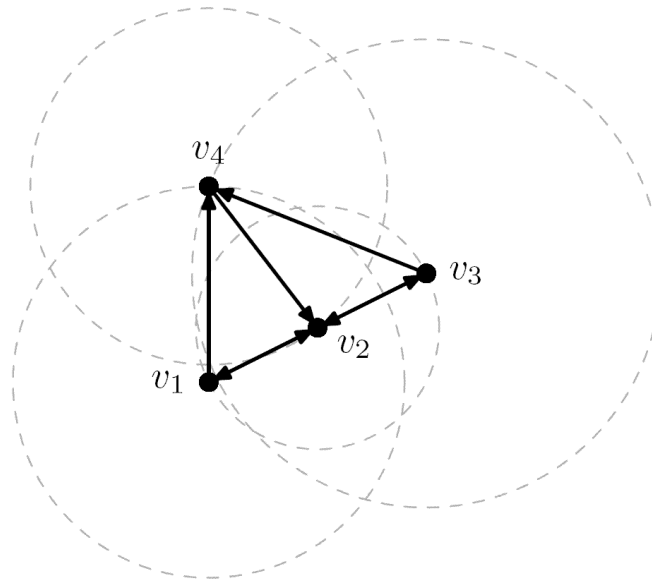
# General Graphs & Mutual Inclusion Graphs



## G: (General Graph)

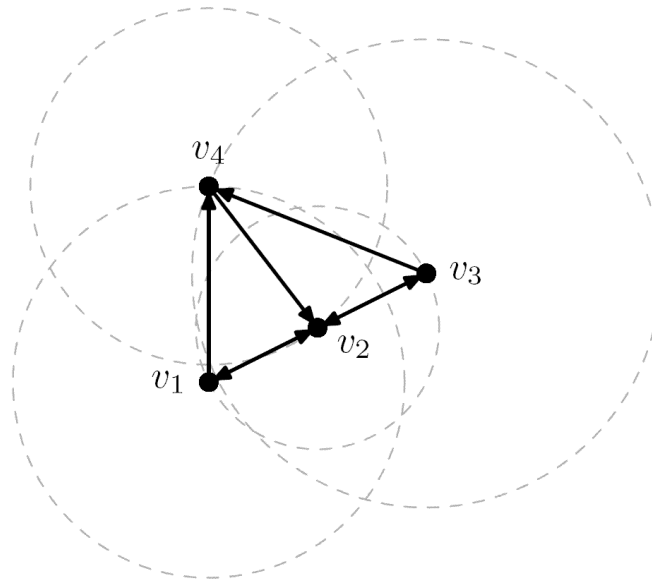
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- a node  $u$  connects to another node  $v$  iff the Euclidean distance between these two nodes is smaller than the transmission range of  $u$
- this model has uni- and bi-directional connections



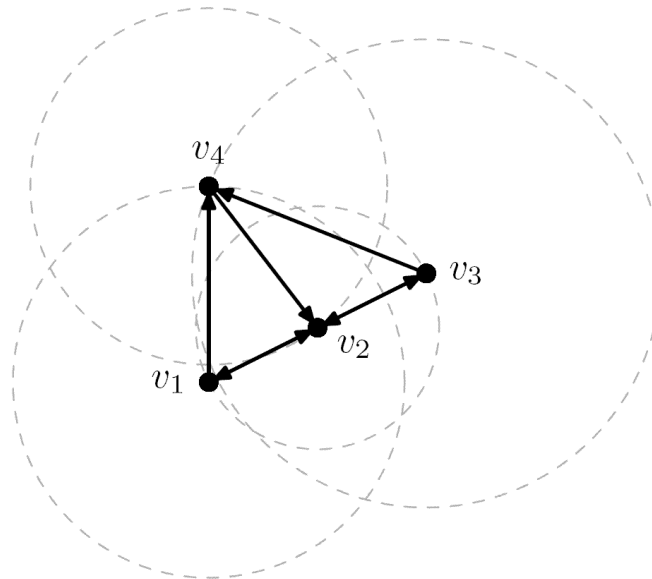
# Reachable Neighbourhood (1)

- in DRNG and DLMST each node has to know its reachable neighbourhood
  - set of nodes that a specific node can reach using its maximal transmission power (eg. for  $v_1$  we get  $v_2$  and  $v_4$ )



## Reachable Neighbourhood (2)

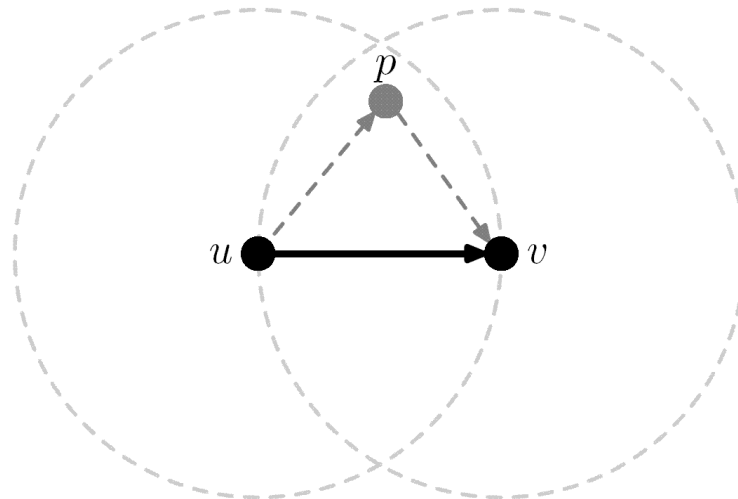
- finding this reachable neighbourhood is difficult since  $v_4$  can't reach  $v_1$ 
  - unfortunately it is not described in the paper how they will manage this in the General Graph



# Directed RNG (Relative Neighbourhood Graph)

- Algorithm:

- collect reachable neighbourhood
- build topology by selecting those nodes from the reachable neighbourhood for which there does not exist a node  $p$  that is closer to  $u$  and  $v$  than  $u$  to  $v$  and  $p$  can reach  $v$ .







# Directed Local MST (Minimum Spanning Tree)

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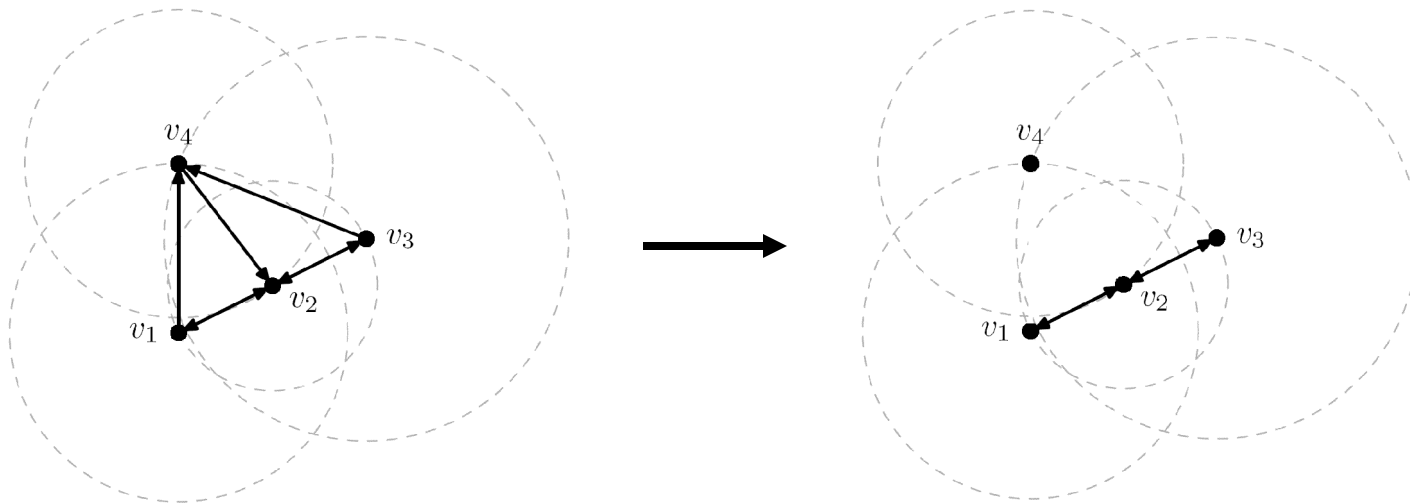
- Algorithm:
  - collect reachable neighbourhood
  - build topology computing a directed MST for each node that spans the reachable neighbourhood of this node and takes on-tree nodes that are one hop away as its neighbours.



## MG: (Mutual Inclusion Graph)

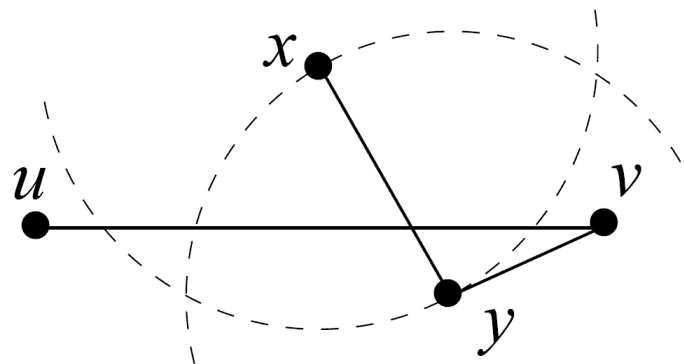
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- two nodes are connected iff they are within the maximum transmission range of each other
- there are only bi-directional links



# Planar Topology

- for any topology control method it is not always possible to create a planar topology while keeping the communication graph connected
  - $u$  is out of the transmission range of  $x$  and  $y$ , while  $v$  is in the transmission range of  $y$  and out of the range of  $x$
  - according to MG, there are only  $xy$ ,  $vy$  and  $uv$  in the graph

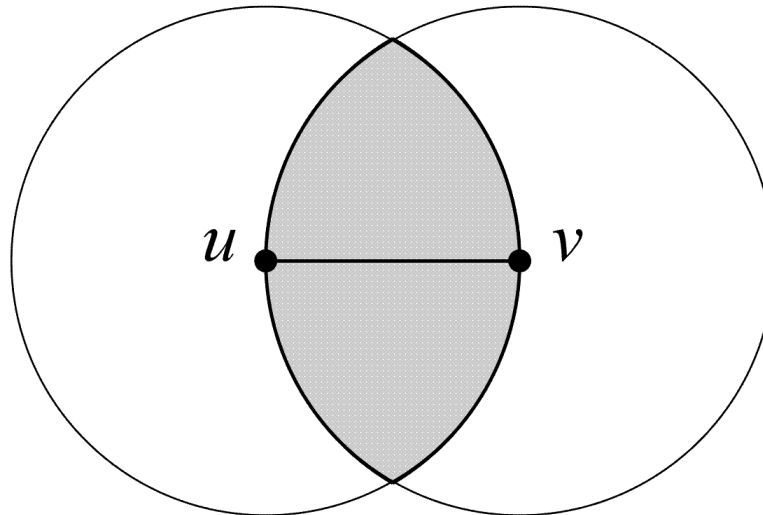




# Sparse Structure

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- based on RNG they found an extension that has bounded number of links  $\rightarrow$  sparse structure
- unfortunately that's not what we want
- we are looking for bounded out-degree





# Idea of Spanners

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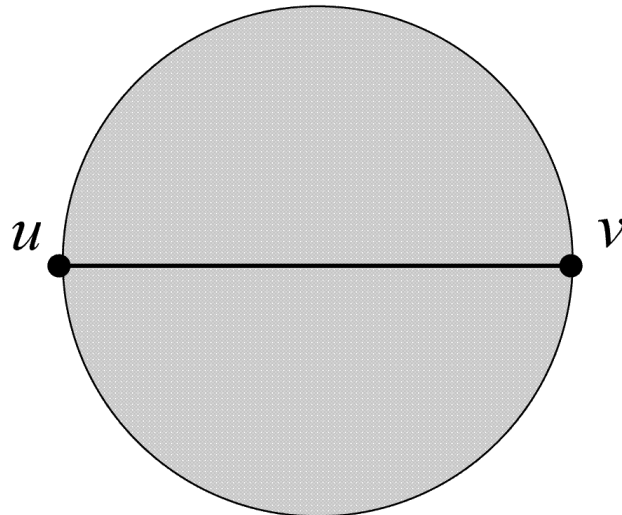
- Given a graph  $G$  and a subgraph  $H$  of  $G$ .
- $H$  is a **t-Length Spanner** of  $G$  if for any two nodes  $u, v \in V(H)$  the shortest path between  $u$  and  $v$  is at most a constant factor  $t$  longer than the shortest path of these two nodes in  $G$ .
- if the weighting function is not the length but the power then we have with the same argumentation a **Power Spanner** instead of a **Length Spanner**



# Power Spanner

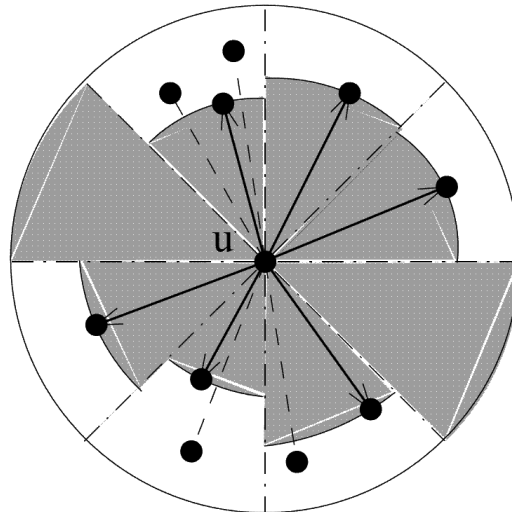
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- based on GG (Gabriel Graph) they found a graph which contains the minimum power consumption path for any two nodes in MG
- we are still looking for bounded out degree



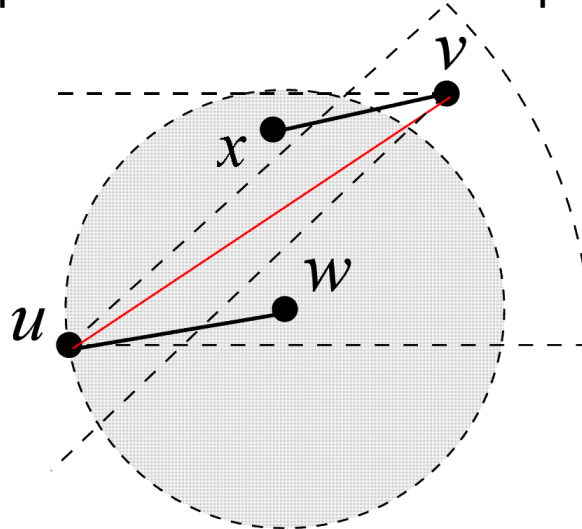
# Degree-Bounded Spanner (1)

- based on Yao Graph
- at each node  $u$ , partition space into  $k$  equal subspaces (= cones) and connect to the nearest node in each cone if there is any



## Degree-Bounded Spanner (2)

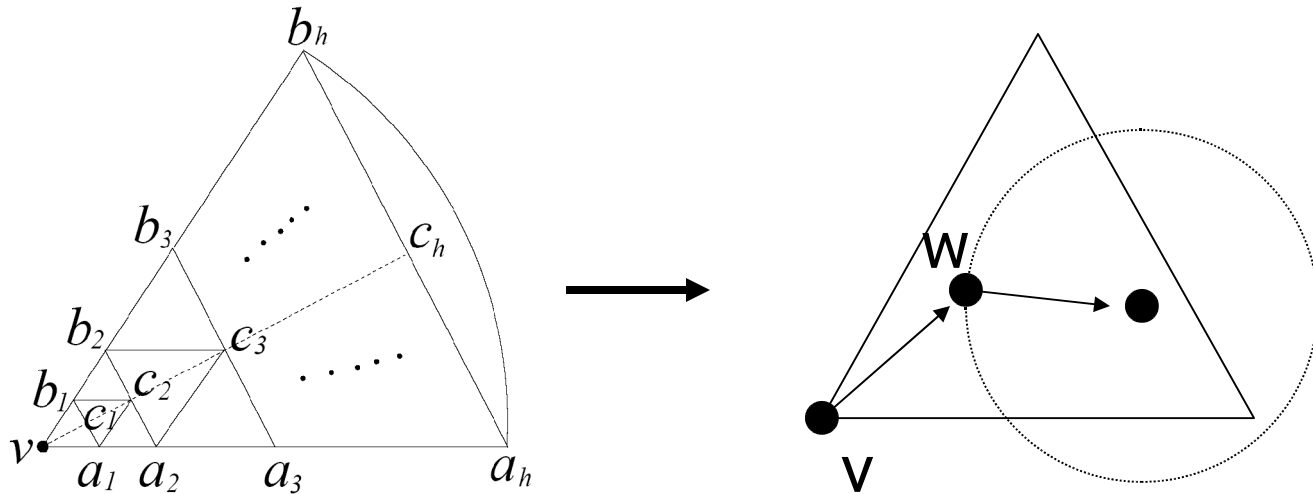
- in a MG model simply selecting the closest incoming neighbour does not guarantee connectivity
  - $v, w$  are in same cone of  $u$  ;  $x, u$  are in same cone of  $v$
  - node  $u$  keeps link  $uw$  and node  $w$  keeps link  $uw$
  - node  $v$  keeps link  $vx$  and node  $x$  keeps link  $xv$ .





# Novel Space Partition

- partition space into  $k$  equal subspaces (= cones)
- divide each cone into constant number of subsets and connect  $v$  to the nearest node  $w$  in each subset
- the algorithm guarantees that all nodes in a subset are connected to node  $w$  in this subset





## EYG<sub>k</sub>(MG): (Extended Yao Graph)

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- has bounded out-degree in  $O(\log_2 q)$
- is a Length- and a Power-Spanner to MG
- is connected if MG is connected
- is bi-directional
- they reach almost optimum since any connected graph will have degree at least  $O(\log_2 q)$
- $q = \max_{v,w} r_v/r_w$  with  $v \in V(\text{MG})$  and  $wv \in \text{EYG}_k(\text{MG})$



# Part 3

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Proof  
&  
Conclusions



# Proof for connectivity in $G_{\text{DLMST}}$ (1)

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- Lemma 1:

For any edge  $(u, v)$  which is only in  $G$  but not in  $G_{\text{DLMST}}$ , there must be a unique path on  $T_u$  from  $u$  to  $v$  in  $G_{\text{DLMST}}$ . Let  $p$  be the last node on this path before  $v$  then we have  $w(p, v) < w(u, v)$ .

$w(u, v)$ : gives any edge in a graph a unique weight

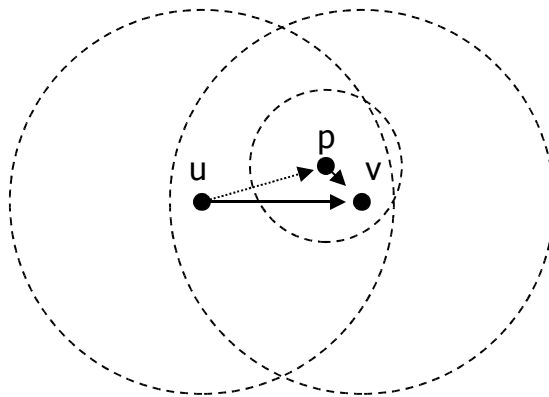
$T_u$ : local MST rooted at node  $u$  containing all reachable nodes of  $u$

$G$ : General Graph

# Proof for connectivity in $G_{\text{DLMST}}$ (2)

- Proof (by contradiction):

Suppose  $w(p, v) > w(u, v)$ , we can construct another directed spanning tree  $T'_u$  rooted at  $u$  with lower weight, by replacing edge  $(p, v)$  with  $(u, v)$  and keeping all the other edges in  $T_u$  unchanged. This contradicts the assumption that  $T_u$  is the local directed MST.





# Proof for connectivity in $G_{\text{DLMST}}$ (1)

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- Lemma 2:

Let  $T$  be the global directed MST of  $G$  rooted at any node  $w \in V(G)$ , then  $E(T) \subseteq E(G_{\text{DLMST}})$ .

- Proof (by contradiction):

For any edge  $(u, v) \in E(T)$  suppose  $(u, v) \notin E(G_{\text{DLMST}})$ . Since  $v$  is on the directed local MST  $T_u$ , there exists a unique path from  $u$  to  $v$  with  $p$  as the last node on this path before  $v$ .

We have  $w(p, v) < w(u, v)$  by Lemma 1. By replacing edge  $(u, v)$  with  $(p, v)$  and keeping all the other edges in  $T$  unchanged, we can construct another global directed spanning tree  $T$  rooted at  $w$  that has lower weight than  $T$ . This contradicts the assumption that  $T$  is the global MST rooted at  $w$ .



## Proof for connectivity in $G_{\text{DLMST}}$ (4)

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- Theorem 1 (Connectivity of  $G_{\text{DLMST}}$ ):

If  $G$  is strongly connected, then  $G_{\text{DLMST}}$  is also strongly connected.

- Proof (by contradiction):

For any two nodes  $u, v \in V(G)$ , there exists a unique global MST  $T$  rooted at  $u$  since  $G$  is strongly connected. Since  $E(T) \subseteq E(G_{\text{DLMST}})$  by Lemma 2, there is a path from  $u$  to  $v$  in  $G_{\text{DLMST}}$ .



# Conclusions: Paper 1 (1)

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- for a General Graph there are two localized topology control algorithms, DLMST and DRNG, which preserve connectivity
- DLMST and DRNG preserve bi-directionality if they are based on a Mutual Inclusion Graph and Addition & Remove operations are applied





## Conclusions: Paper 1 (2)

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- DLMST has a bounded out-degree while DRNG may be unbounded
- there is no description of how exactly they find the reachable neighbourhood
  - it is more like a theoretical and mathematical work showing the general possibility for building such topologies based on a General Graph



## Conclusions: Paper 2

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- $EYG_k(MG)$  has a stricter bound on the out-degree than DLMST and guarantees better characteristics
- Length- and Power-Spanner to MG
- they reach almost optimum since any connected graph will have degree at least  $O(\log_2 q)$



Questions?

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