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Principles of Distributed Computing
Exercise 9

1 Pancake Networks

In the lecture, you have encountered several different graphs as underlying network structures for peer-to-peer (P2P) networks. Here, we will look at another prominent example, the *Pancake graph*[1] P_n.

The pancake graph P_n is defined as follows: The vertex set is

$$V(P_n) = \{(v_1, v_2, \ldots, v_n) \mid v_i \in \{1, \ldots, n\} \text{ and } v_i \neq v_j \ \forall i \neq j\}\,.$$

In other words, $V(P_n) = S_n$, the group of all permutations on n elements. There exists an edge of dimension i for $2 \leq i \leq n$ when

$$e_i = \{(v_1, \ldots, v_i \ldots, v_n), (w_1, \ldots, w_i, \ldots, w_n)\} \in E(P_n) \Longleftrightarrow \\ w_j = v_{i-j+1} \text{ for } 1 \leq j \leq i \text{ and } w_j = v_j \text{ for } i < j \leq n,$$

or, we can say that an edge e_i represents a *prefix reversal*

$$(v_1, \ldots, v_i, v_{i+1}, \ldots, v_n) \longleftrightarrow (v_i, \ldots, v_1, v_{i+1}, \ldots, v_n). \quad (1)$$

For the following questions, where appropriate, give your answers in terms of $N := |V(P_n)|$ (approximately), the number of vertices, as well as n.

a) Draw (nicely!) P_n for $n = 2, 3, 4$. Try to describe a pattern for drawing P_n for any n.

b) What is the degree of each vertex in P_n?

c) Can you give bounds on the diameter $D(P_n)$ of the pancake network?

d) Show that P_n is Hamiltonian for $n \geq 3$.

e) How can the pancake graph be used to implement a *distributed hash table* (DHT)? In other words, where are files, indexed by bitstrings of a certain length b, stored in the pancake graph, and how can these files be looked up (given the corresponding bitstring)?[2]

The pancake graph has been proposed for P2P networks partly because of the properties analyzed in this exercise.

[1] A well-known paper about pancake graphs was originally written in 1976 by a 21 year old college dropout (later to become a famous entrepreneur) and a Ph.D. student (later to become a famous scholar):
W. Gates and C. Papadimitriou. Bounds for Sorting by Prefix Reversal. *Discrete Math.*, 27:4757, 1979.

[2] You can ignore churn in this exercise.