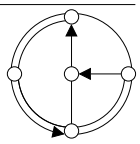
ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

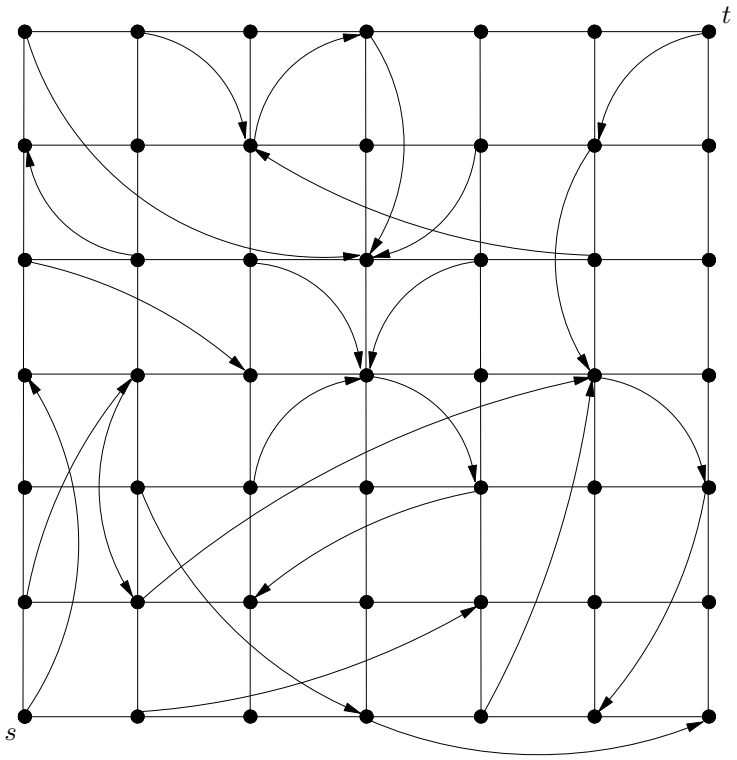


SS 2007Prof. R. Wattenhofer / Prof. P. Widmayer / Prof. S. Suri / D. Bilò/ T. Locher / Y. A. Oswald

Principles of Distributed Computing
Exercise 4

1 Greedy algorithm with lookahead

Consider a 2-dimensional Small-World network with $r = 2$ and $p = q = 1$. At any current node u, algorithm `Greedy` forwards the message to a neighbor of u that is as close to the target t as possible. The algorithm `Look(`k`)`, where $k > 1$ is an integer, is an extension of algorithm `Greedy`, and uses a lookahead of k steps as follows. At a current node u, `Look(`k`)` considers the set U of all the nodes within lattice distance $k - 1$ from u, as well as all the nodes reachable from U via either a local or a long-range contact. We call this set $N^k(u)$. From node u, algorithm `Look(`k`)` forwards the message to a node in $N^k(u)$ that is as close to the target t as possible. Notice that this may require traversing up to k arcs. Answer to the following questions



a) In the Small-World network above, all local contacts (grid arcs) are bi-directed while the directed long-range contacts are only shown when different from the local contacts. Construct an $s - t$ path for `Look(2)`.

b) What is the expected delivery time of algorithm `Look(2)`?

c) What is the expected delivery time of algorithm `Look(`$\log n$`)`?

2 3-dimensional Small-World networks

Consider the Small-World network for a 3-dimensional grid, where each node u has its 6 neighbors as local contacts, and long-range contacts are generated with the proportional long-range contact probabilities

$$\Pr[u \text{ has a long-range contact } x] = \frac{d(u,x)^{-3}}{\sum_{v \neq u} d(u,v)^{-3}}$$

a) Show that the normalizing constant $\sum_{v \neq u} d(u,v)^{-3}$ for this probability distribution is at most $k_1 \ln(k_2 n)$ for some constants k_1, k_2.

b) What is the expected delivery time of algorithm `Greedy` (defined in Exercise 1) for the case $p = q = 1$?